SOFiSTiK

Benchmark Example No. 39

Deflection of Reinforced Concrete Beam with Creep and Shrinkage

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VERiFiCATiON
DCE-EN39 Deflection of Reinforced Concrete Beam with Creep and Shrinkage

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Front Cover

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	BEAM
Input file(s):	einfeldbalken_verformung_zilch390_1.dat

1 Problem Description

The problem consists of a 1 m' wide slab strip, reinforced for bending in lower layer, as shown in Fig. 1. The deflection with effects of creep and shrinkage is calculated with the BEAM module and compared with the reference example Zilch and Zehetmaier [1, Example 10.2-10.4].

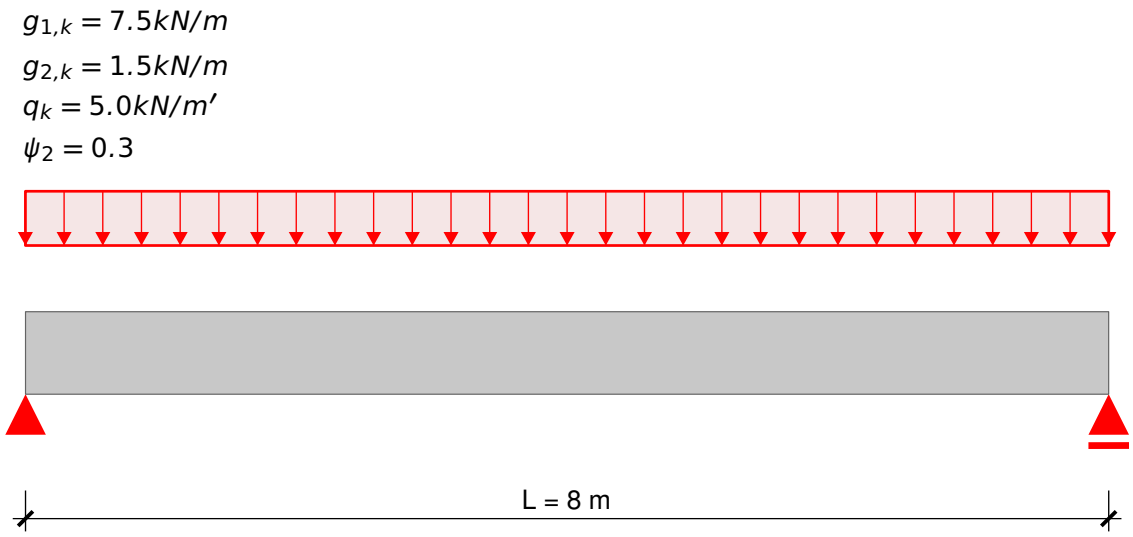

Figure 1: System and Loads

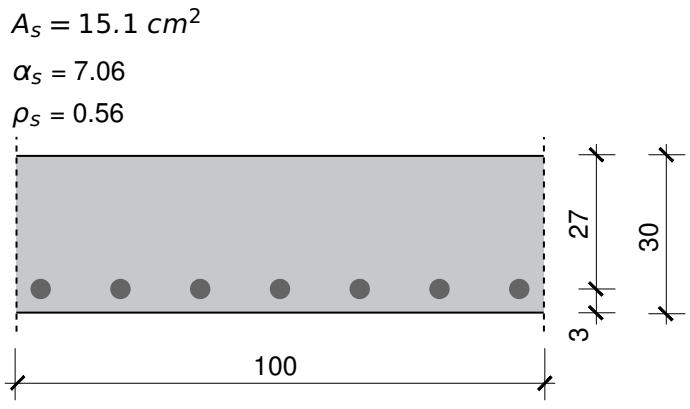

Figure 2: Cross-Section B/H=400/800 mm

2 Reference Solution

As part of the verifications in the serviceability limit state, the verification of the limitation of deformation is an important aspect in the assessment of serviceability. According to EC2, the deformations and

deflections of a reinforced concrete component must be limited, for example to prevent possible damage to neighboring components or to ensure that the usability of a component is not restricted.

The deformations of reinforced concrete components are largely dependent on the building material concrete and its ability to react to tensile forces. In the composite material reinforced concrete, the reinforcement supports the ability to bear the tensile forces - the concrete can only bear these to a limited extent. As soon as the tensile forces in the edge fibers of the concrete exceed its average tensile strength, cracks form in the concrete and consequently the component deforms. The deformations are significantly influenced by states I and II of the concrete.

3 Model and Results

Table 1: Model Properties

Material Properties	Geometric Properties
C 30/35	$h = 30.0\ cm$
B 500B	$b = 100\ cm$
	$d = 27.0\ cm$
	$A_{s,lower} = 15.1\ cm^2$

Table 2: Results

Deflection [mm]	SOF. BEAM	Ref. Zilch and Zehetmaier [1, Example 10.2-10.4]
For t $= \infty$		
w_{total}	59.778	60.00
$w^{I}_{creep+loading}$	22.225	22.30
$w^{II}_{creep+loading}$	47.044	47.20
$k^{I}_{shrinkage}$	0.824	0.825
$k^{II}_{shrinkage}$	2.673	2.666

4 Design Process

The reinforcement amount and distribution are constant over the length of the member; accordingly EI^I, EI^{II} and the M-κ relationship apply to the entire member. The quasi-permanent actions are:

$$(g+q)_{d,perm} = g_{1k} + g_{2k} + \Psi_2 \cdot q_k$$

$$(g+q)_{d,perm} = 7.5 + 1.5 + 0.3 \cdot 5 = 10.5\ kN/m$$

Deflection w^I and w^{II}

For the uncracked state, the bending stiffness is constant over the entire member length, therefore w^I can be calculated using:

$$\delta W^{(a)} = \delta W^{(i)}$$

$$\Rightarrow w = \frac{5}{384} \cdot \frac{q \cdot l^4}{E_c \cdot E_i}$$

Since the height of the compression zone and and thus the bending stiffness in pure state II for cross-sections subjected to bending only is independent of the load, the same applies to w_{II} . For the uncracked beam applies:

$$EI^I \approx E_{cm} \cdot I_c = 28300 \cdot \frac{1.0 \cdot 0.3^2}{12}$$

$EI^I \approx 63.7 MNm^2$ (Gross Cross-Section)

$$w^I = \frac{5}{384} \cdot \frac{(g+q)_{perm} \cdot l^4}{EI^I}$$

$$w^I = \frac{5}{384} \cdot \frac{10.5 \cdot 10^{-3} \cdot 8^4}{63.7}$$

$$w^I = 8.8 \cdot 10^{-3}\ m = 8.8\ mm$$

In pure state II applies:

$$\xi = \alpha_s \cdot \rho_s + \sqrt{(\alpha_s \cdot \rho_s)^2 + 2\alpha_s \rho_{s1}} = 0.244$$

$$\rightarrow x^{II} = \xi \cdot d = 0.066\ m$$

$$z^{II} = d - \frac{x^{II}}{3} = 0.248\ m$$

$$\rightarrow EI^{II} = E_s \cdot A_s \cdot z^{II} \cdot (d - x^{II}) = 15.3\ MNm^2$$

$$\rightarrow w^{II} = \frac{5}{384} \cdot \frac{(g+q)_{perm} \cdot l^4}{EI^{II}}$$

$$w^{II} = 36.7 \cdot 10^{-3}\ m = 36.7\ mm$$

The bending stiffness of the pure state II is only 24% of the uncracked cross-section. Accordingly, the deflection is $1/0.24 = 4.2$ times higher.

In Eurocode 2: Members which are not expected to be loaded above

the level which would cause the tensile strength of the concrete to be exceeded anywhere within the member should be considered to be uncracked. Members which are expected to crack, but may not be fully cracked, will behave in a manner intermediate between the uncracked and fully cracked conditions and, for members subjected mainly to flexure, an adequate prediction of behaviour is given by expression:

$$\alpha = \zeta \cdot \alpha_{II} + (1 - \zeta) \cdot \alpha_I$$

Table 3: Calculating the deflection with ζ

x_i	M_i	κ_i^I	κ_i^{II}	σ_{s2}	ζ_i	$\kappa_{m,i}$	δM_i	f_i
0.0	0.00	0.000	0.000	0.0	0.000	0.000	0.0	1
0.2	8.19	0.129	0.537	21.9	0.000	0.129	0.1	4
0.4	15.96	0.251	1.046	42.7	0.000	0.251	0.2	2
0.6	23.31	0.366	1.527	62.3	0.000	0.366	0.3	4
0.8	30.24	0.475	1.981	80.9	0.000	0.475	0.4	2
1.0	36.75	0.577	2.408	98.3	0.000	0.577	0.5	4
1.2	42.84	0.673	2.807	114.5	0.000	0.673	0.6	2
1.4	48.51	0.762	3.178	129.7	0.455	1.860	0.7	4
1.6	53.76	0.844	3.522	143.7	0.718	2.766	0.8	2
1.8	58.59	0.920	3.838	156.7	0.841	3.374	0.9	4
2.0	63.00	0.989	4.127	168.4	0.852	3.663	1.0	2
2.2	66.99	1.052	4.389	179.1	0.861	3.924	1.1	4
2.4	70.56	1.108	4.623	188.7	0.868	4.158	1.2	2
2.6	73.71	1.157	4.829	197.1	0.873	4.364	1.3	4
2.8	76.44	1.200	5.008	204.4	0.878	4.543	1.4	2
3.0	78.75	1.236	5.159	210.6	0.882	4.694	1.5	4
3.2	80.64	1.266	5.283	215.6	0.884	4.818	1.6	2
3.4	82.11	1.289	5.379	219.5	0.886	4.915	1.7	4
3.6	83.16	1.306	5.448	222.3	0.888	4.983	1.8	2
3.8	83.79	1.315	5.489	224.0	0.889	5.025	1.9	4
4.0	84.00	1.319	5.503	224.6	0.889	5.038	2.0	1

For the integration of w, the module BEAM is using *trapezoidal* instead of the *approximation* method.

$$w(t_0) = 2 \cdot \int_0^{l/2} \delta M \cdot \kappa_m \, \mathrm{dx} = 2 \cdot \kappa_{m,i} \cdot \delta M_i \cdot f_i \cdot \Delta x \cdot \frac{1}{3}$$

$$w(t_0) = 31.7 \; mm$$

The calculation follows Eurocode 2 and the relation of the creep coefficient to the tangent modulus is calculated by multiplication of ρ by α_i in $E_{c,eff}$. Final creep coefficient, final shrinkage and α_i are (internal component, $t_0 = 28$ days):

$\rho(\infty, t_0) = 2.23$

$\epsilon_{cs,\infty} = -0.63$ ‰

$\alpha_i = 0.886$

Creep

The effective modulus of elasticity can be used to calculate the bending stiffnesses in state I and in pure state II (ideal cross-section values used for state I). The following applies for $t \to \infty$:

$$E_{c,eff}(t) = \frac{E_{cm}}{1 + \alpha_i \cdot \rho} = 9503\ N/mm^2$$

$\to \alpha_{s,eff} = \alpha_s(t) = 21.0$

$EI^I(t) = E_{c,eff}(t) \cdot I_1 = 25.1\ MNm^2$ (Reduction about 63%)

$$\xi(t) = -\alpha_s(t) \cdot \rho_{s1} + \sqrt{(\alpha_s(t) \cdot \rho_{s1})^2 + 2 \cdot \alpha_s(t) \cdot \rho_{s1}}$$

$\xi(t) = 0.381 \to x(t) = 0.103\ m$

$EI^{II}(t) = E_s \cdot A_s \cdot z(t) \cdot (d - x(t))$

$EI^{II}(t) = 11.9\ MNm^2$ (Reduction about 22%)

The bending deformations in state I or in pure state II increase in proportion to the bending stiffnesses for $t = 0$ and $t \to \infty$:

$$w^I(t) = w^I(t_0) \cdot \frac{EI^I(t_0)}{EI^I(t)}$$

$$w^I(t) = 8.8 \cdot \frac{63.7}{25.1} = 22.3\ mm$$

$$w^{II}(t) = w^{II}(t_0) \cdot \frac{EI^{II}(t_0)}{EI^{II}(t)}$$

$$w^{II}(t) = 36.7 \cdot \frac{15.3}{11.9} = 47.2\ mm$$

Shrinkage

The shrinkage of concrete leads to deviation of the reinforcement's center from the center of gravity in both the uncracked and cracked cross-section. The calculation is carried out using the equivalent force N_{cs}.

$N_{cs} = \epsilon_{cs,\infty} \cdot E_s \cdot A_s = 0.190\ MN = 190\ kN$

In state I, the distance of the centre of gravity of the reinforcement from the centre of gravity of the ideal cross-section is $z_s = 0.109\ m$. The shrinkage curvature:

$$\kappa^I_{cs} = \frac{M_{cs}}{EI^I(t)} = \frac{0.190 \cdot 0.109}{25.1} = 8.3 \cdot 10^{-4}\ 1/m$$

In pure state II, the null line is the mechanical centre of gravity of the cross-section; the curvature thus follows:

$$z_s = d - x(t) = 0.27 - 0.103 = 0.167$$

$$\kappa_{cs}^{II} = \frac{M_{cs}}{EI^{II}(t)} = \frac{0.190 \cdot 0.167}{11.9} = 26.7 \cdot 10^{*-4}\ 1/m$$

The shrinkage curvature taking into account the tensile stiffening is determined for the cracked area is determined by weighting over the distribution coefficient ζ:

$$\kappa_{cs,m}^{II} = \zeta \cdot \kappa_{cs}^{II} + (1 - \zeta) \cdot \kappa_{cs}^{I}$$

$\beta = 1.0$ (short-term loading)
$\beta = 0.5$ (permanent/recurrent loading)

$$\zeta = 1 - \beta \cdot \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$$

$\zeta = 0.889$ (see Table 3)

$$\kappa_m = \zeta \cdot \kappa_s^{II} + (1 - \zeta) \cdot \kappa_s^{I}$$

$$\kappa_m = 0.889 \cdot 2.673 + (1 - 0.889) \cdot 0.856 = 2.471$$

$$w_{shrinkage} = \frac{5 \cdot l^2}{48} \cdot \kappa_m$$

$$w_{shrinkage} = \frac{5 \cdot 8^2}{48} \cdot 2.471$$

$$w_{shrinkage} = 16.47\ mm$$

The deflection in the middle of the field is again calculated by integrating the sum of the load-dependent- and shrinkage curvature using the principle of virtual forces. The resulting deflection in the middle of the field for $t \to \infty$ is:

$$w(t) = 60\ mm,$$

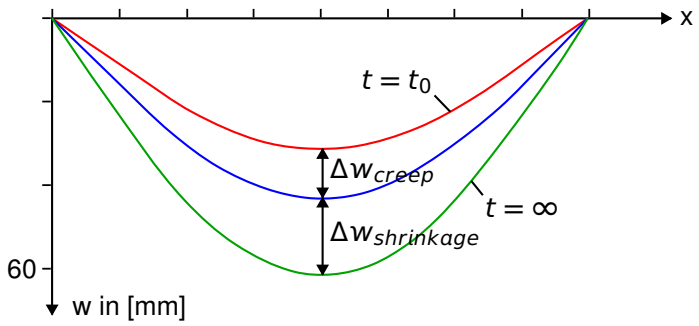

Figure 3: System and Loads

The deflection with the effects of creep and shrinkage is approximately twice the deflection at t_0. Fig. 3 shows the individual proportions of the cross-sectional curvatures and the resulting deflection in the middle of the field are shown.

5 Conclusion

In this example, the beam deflection calculated in BEAM is verified. It has been shown that the deflection is determined accurately.

6 Literature

[1] K. Zilch and G. Zehetmaier. *Bemessung im konstruktiven Betonbau*. 2nd. Springer, 2010.