SOFiSTiK

Benchmark Example No. 38

Deflection of Reinforced Concrete Beam

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VERiFiCATiON
DCE-EN38 Deflection of Reinforced Concrete Beam

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SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	BEAM
Input file(s):	beam_deflection.dat

1 Problem Description

The problem consists of a rectangular section, reinforced for bending in lower layer, as shown in Fig. 1. The deflection of the beam is calculated with the BEAM module and compared with a non-linear analysis.

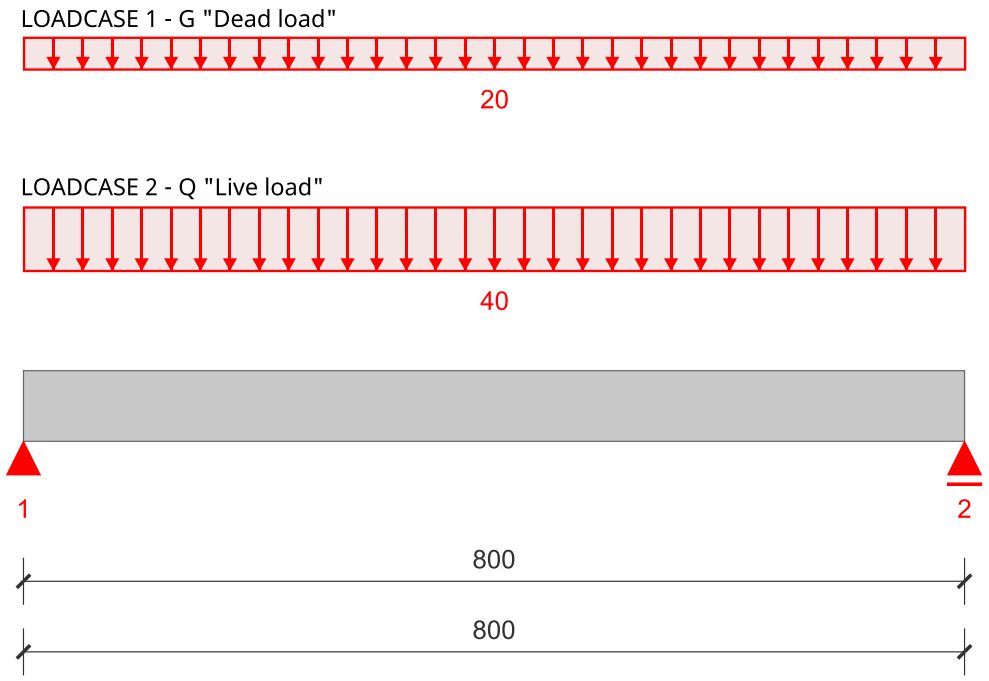

Figure 1: System and Loads

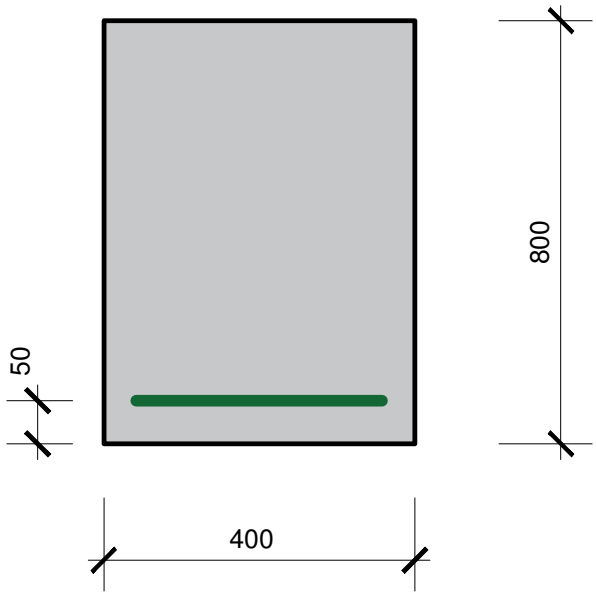

Figure 2: Cross-Section B/H=400/800 mm

2 Reference Solution

As part of the verifications in the serviceability limit state, the verification of the limitation of deformation is an important aspect in the assessment of serviceability. According to EC2, the deformations and deflections of a reinforced concrete component must be limited, for example to prevent possible damage to neighboring components or to ensure that the usability of a component is not restricted.

The deformations of reinforced concrete components are largely dependent on the building material concrete and its ability to react to tensile forces. In the composite material reinforced concrete, the reinforcement supports the ability to bear the tensile forces - the concrete can only bear these to a limited extent. As soon as the tensile forces in the edge fibers of the concrete exceed its average tensile strength, cracks form in the concrete and consequently the component deforms. The deformations are significantly influenced by states I and II of the concrete.

The BEAM calculation is compared with a non-linear analysis using the modules ASE, STAR2 and also with a hand-calculation according to EN 1992-1-1.

3 Model and Results

Table 1: Model Properties

Material Properties	Geometric Properties
C 25/30	$h = 80.0\ cm$
B 500B	$b = 40\ cm$
	$d = 75.0\ cm$
	$A_{s,lower} = 31.45\ cm^2$

Table 2: Results

	SOF. BEAM	SOF. ASE (Beams)	SOF. ASE (Quads)	SOF. STAR2 (Beams)	EN 1992-1-1 (Ch. 7.4.3)
$w\,[mm]$	11.041	11.794	11.719	11.803	11.276
$w^{I}\,[mm]$	4.251	–	–	–	4.764
$w^{II}\,[mm]$	11.513	–	–	–	11.555
ζ	0.959	–	–	–	0.959

4 Design Process

According to the general principles of mechanics, vatures of a component can be determined as a function of the load and the stiffness of the cross-section:

$$k = \frac{1}{r} = \frac{M}{EI}$$

The deflection from the curvature is calculated by double integral:

$$w(x) = \int \int \frac{M(x)}{EI(x)} \mathrm{dx}$$

The dependencies of the deflections are therefore in relation to the system length, load and stiffness. Assuming a moment curve affine to the curvature curve, the double integral of the curvature can be converted to the following expression using the principle of virtual forces:

$$w(x) = \int \frac{\overline{M}(x) \cdot M(x)}{EI(x)} \mathrm{dx}$$

$$w(x) = \int \left(\overline{M}(x) \cdot \kappa\right) \mathrm{dx} \approx k \cdot \kappa \cdot l^2 = w\left(\frac{l}{2}\right)$$

k describes a coefficient for observing the moment curve and can be derived from the integral tables or taken directly from the literature.

The aim of the approximation method according to EC2 is essentially a simplified suggestion of the curvature of a reinforced concrete component and provides the mean expected curvature of the component under consideration, taking into account the crack distribution coefficient. The maximum expected curvatures of state I and state II are calculated.

Assuming a constant uniform line load and stiffness - i.e. neglecting states I and II - the numerical integration can be simplified to:

$$w(l/2) = \frac{5}{384} \cdot \frac{p_k \cdot l^4}{EI}$$

With this simplification, the deflection can be calculated separately for states I and II and then calculated analogously to the approximation method, assuming the crack distribution coefficient for an expected mean curvature.

The deviations between the approximation method according to EC2 and the numerical integration amount to approximately 4%; the calculation according to the approximation method is accurate enough for construction practice. We will use this approach for the hand calculation to simplify our verification.

In I state:

$$EI^I \approx E_{cm} \cdot Ic$$

$$EI = 31476 \, \frac{MN}{m^2} \cdot 1.707 \cdot 10^{-2} m^4$$

$$EI = 537.3 \frac{MN}{m^2}$$

Calculating the deflection in state I.

$$w^I = \frac{5}{384} \cdot \frac{(g + q \cdot \psi)_{perm} \cdot l^4}{EI}$$

$$w^I = \frac{5}{384} \cdot \frac{(20 + 40 \cdot 0.7)_{perm} \cdot 10^{-3} \cdot 8^4}{537.3}$$

$$w^I = 0.00476\ mm = 4.764\ mm$$

In II state:

Since the compression zone height and thus the bending stiffness in pure state II is independent of the load for only bent cross-sections, the upper equations can also be applied for w^{II}.

$$d = h - c_v$$

$$d = 800 - 50 = 750\ mm$$

$$\xi = -\alpha \cdot \rho_{s1} + \sqrt{(\alpha_s \cdot \rho_{s1})^2 + 2\alpha_s \cdot \rho_{s1}}$$

$$\alpha_s = \frac{E_s}{E_c} = \frac{200000}{31476} = 6.354$$

$$\rho_l = \frac{A_{su}}{b \cdot d} = \frac{31.45 \cdot 10^{-4}}{0.4 \cdot 0.75} = 0.0104$$

$$\xi = -6.354 \cdot 0.0104 + \sqrt{(6.354 \cdot 0.0104)^2 + 2 \cdot 6.354 \cdot 0.0104}$$

$$\xi = 0.303418$$

$$x^{II} = \xi \cdot d$$

$$x^{II} = 0.303418 \cdot 750 = 227.5635\ mm$$

$$z^{II} = d - \frac{x^{II}}{3}$$

$$z^{II} = 750 - \frac{227.5635}{3} = 674.1455\ cmm$$

$$EI^{II} = E_s \cdot A_s \cdot z^{II} \cdot (d - x^{II})$$

$$EI^{II} = 200000 \cdot 31.45 \cdot 10^{-4} \cdot 0.6741455 \cdot (0.75 - 0.2275635)$$

$$EI^{II} = 200000 \cdot 31.45 \cdot 10^{-4} \cdot 0.6741455 \cdot (0.75 - 0.2275635)$$

$$EI^{II} = 221.53$$

$$w^{II} = \frac{5}{384} \cdot \frac{(g + q)_{perm} \cdot l^4}{EI^{II}}$$

$$w^{II} = \frac{5}{384} \cdot \frac{(20 + 40 \cdot 0.7)_{perm} \cdot 10^{-3} \cdot 8^4}{221.53}$$

$$w^{II} = 0.011555\ m = 11.555\ mm$$

The aim of the approximation method according to EC2 is essentially a simplified suggestion of the curvature of a reinforced concrete component and provides the mean expected curvature of the component under consideration, taking into account the crack distribution coefficient.

The crack distribution coefficient is required to calculate the mean curvature. The crack distribution coefficient expresses the distribution of state I and state II over the component axis and can be calculated using the following approach:

$$\zeta = 1 - \beta \cdot \left(\frac{\sigma_{sr}}{\sigma_s} \right)^2$$

Because we have only bending moments in our example, following equation can be used as alternative to simplify the hand calculation:

$\beta = 1.0$ for a single short-term loading;
$\beta = 0.5$ for sustained loads or many cycles of repeated loading.
EN 1992-1-1, 7.4.3 (3)

$$\zeta = 1 - \beta \cdot \left(\frac{M_{cr}}{M} \right)^2$$

$$M_{cr} = W \cdot f_{ctm}$$

$$M_{cr} = 0.042675\ m^3 \cdot 2.56\ \frac{MN}{m^2} = 0.109248\ MN$$

$$M_{cr} = 109.248\ kNm$$

$$\zeta = 1 - 0.5 \cdot \left(\frac{M_{cr}}{M} \right)^2$$

$$\zeta = 1 - 0.5 \cdot \left(\frac{109.248}{384.00} \right)^2 = 0.959$$

Members which are expected to crack, but may not be fully cracked, will behave in a manner intermediate between the uncracked and fully cracked conditions and, for members subjected mainly to flexure, an adequate prediction of behaviour is given by expression:

As a simplification, α may also be taken as a deflection. EN 1992-1-1, 7.4.3 (3)

$$w = \zeta \cdot w^{II} + (1 - \zeta) \cdot w_I$$

$$w = 0.959 \cdot 11.555 + (1 - 0.959) \cdot 4.764$$

$$w = 11.276\ mm$$

5 Conclusion

In this example, the beam deflection calculated in BEAM is verified. It has been shown that the deflection is determined accurately.

Both, the non-linear and manual calculation provide similar results as BEAM with minor deviations.