Benchmark Example No. 59

SDOF System with Elasto-plastic Worklaw and Sine Pulse Excitaiton

SOFiSTiK | 2026

VERiFiCATiON
BE59 SDOF System with Elasto-plastic Worklaw and Sine Pulse Excitaiton

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Front Cover

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Overview	
Element Type(s):	SPRI, DAMP
Analysis Type(s):	DYN
Procedure(s):	TSTP, MNL
Topic(s):	
Module(s):	FEABENCH
Input file(s):	plastic_spring.dat

1 Problem Description

This problem involves a damped single-degree-of-freedom (SDOF) system (see Figure 1), with a non-linear elasto-plastic spring, as illustrated in Figure 2 (a). The system is subjected to a half-sine impulse force, as shown in Figure 2 (b). The objective is to verify the structural response, computed using the Newmark time integration method with constant acceleration, and the Newton-Raphson algorithm to account for physical nonlinearity.

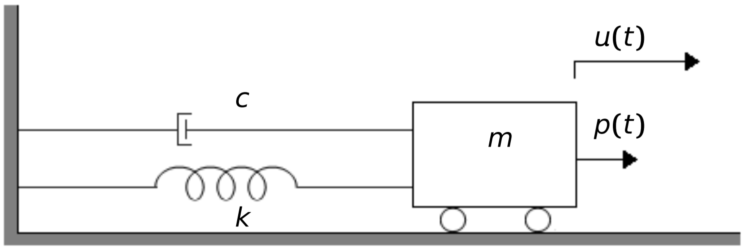
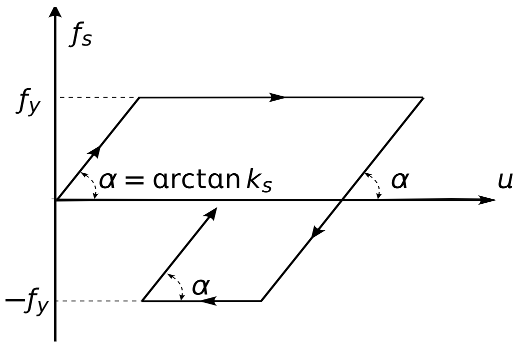
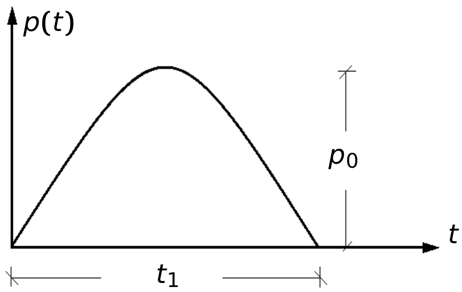


Figure 1: Problem Description



(a) Elasto-plastic worklaw



(b) Half Sine Impulse Loading

Figure 2: Spring and Excitation Schematic

2 Parameters and Reference Solution

The model and loading parameters are provided in Table 2. The reference solution is based on a numerical example from [1] (Example 5.5), with units converted to the metric system. The numerical parameters for time integration follow Newmark's constant acceleration scheme, using a time step of $dt = 0.1\ sec$ and a total integration time of $t = 1.0\ sec$.

Table 1: Model Properties

Model Properties	Excitation Properties
$m = 45.594\,t$	$t_1 = 1.2\,sec$
$c = 28.647\,kN/m$	$p_0 = 1\,kN$
$f_y = 36\,kN$	
$k_s = 1800\,kN/m$	

3 Model and Results

The dynamic analysis in SOFiSTiK is carried out using the FEABENCH solver. Damping is modeled as Rayleigh damping through the mass-proportional coefficient. The spring behavior follows an elasto-plastic material law with isotropic hardening. Table 2 presents the displacements u and the corresponding spring forces f_s, which are compared to reference values at each time step. Figure 3 shows the displacement time history of the SDOF system, while Figure 4 illustrates the force-displacement diagram. The results show good agreement with the reference solution. The force-displacement diagram captures both the loading and unloading branches.

Table 2: Results

t [sec]	u_{ref} [cm]	u_{sof} [cm]	$\lvert e_r \rvert$ [%]	$f_{s,\text{ref}}$ [kN]	$f_{s,\text{sof}}$ [kN]	$\lvert e_r \rvert$ [%]
0.1	0.1213	0.1213	0.00	2.1833	2.1833	0.00
0.2	0.6462	0.6462	0.01	11.6309	11.6310	0.01
0.3	1.7002	1.7002	0.00	30.6034	30.6036	0.00
0.4	3.1034	3.1034	0.00	36.0000	36.0000	0.00
0.5	4.5434	4.5435	0.00	36.0000	36.0000	0.00
0.6	5.6262	5.6262	0.00	36.0000	36.0000	0.00
0.7	6.0103	6.0104	0.00	36.0000	36.0000	0.00
0.8	5.6409	5.6410	0.00	29.3505	29.3509	0.00
0.9	4.7224	4.7225	0.00	12.8176	12.8182	0.00
1.0	3.6062	3.6064	0.00	-7.2737	-7.2732	0.00

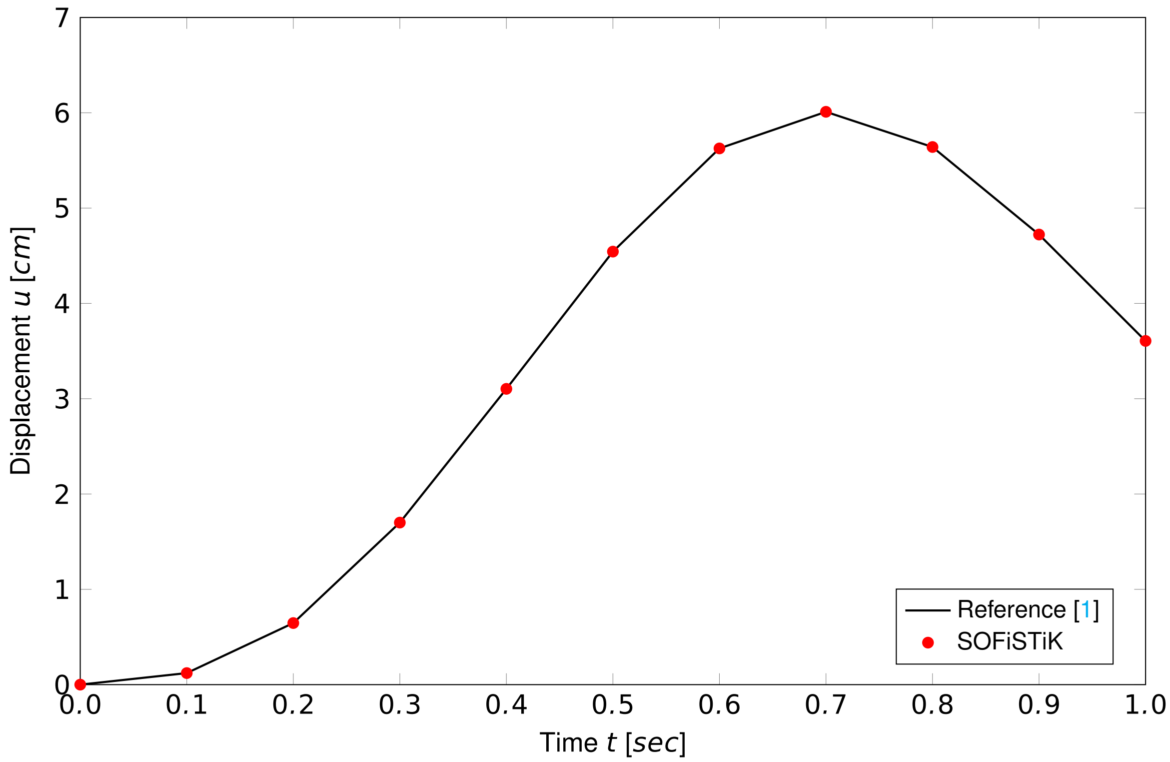


Figure 3: Displacement response of a SDOF system with elastoplastic worklaw

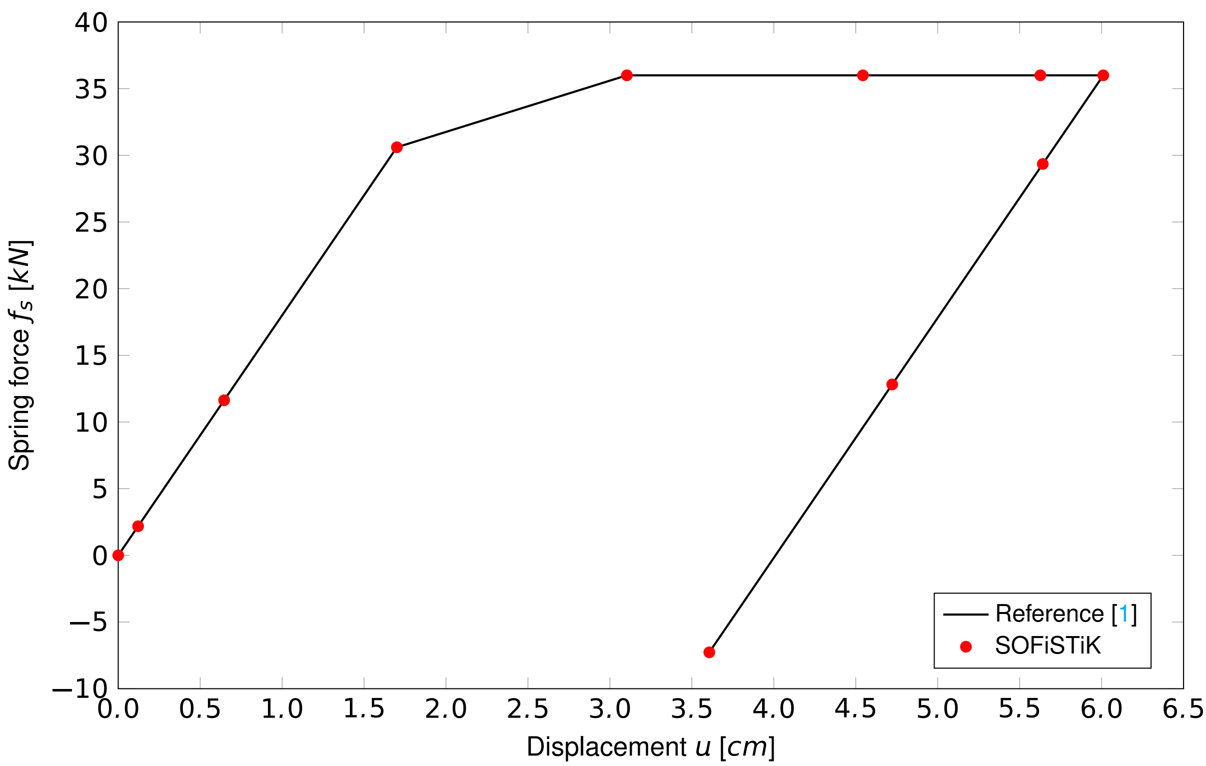


Figure 4: Force-displacement response of a SDOF system with elastoplastic worklaw

4 Conclusion

This example investigates the response of a damped SDOF system with a nonlinear elasto-plastic spring exhibiting isotropic hardening. The results serve to verify the implementation of the time integration scheme in the presence of material nonlinearity. The system's behavior is captured accurately, thereby verifying the numerical implementation of Newmark's time integration and the nonlinear Newton-Raphson algorithms.

5 Literature

[1] A. K. Chopra. *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. Prentice Hall, 1995.