SOFiSTiK

Benchmark Example No. 7

Design of a T-section for Shear

SOFiSTiK | 2024

VERiFiCATiON
DCE-EN7 Design of a T-section for Shear

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Front Cover

6th Street Viaduct, Los Angeles   Photo: Tobias Petschke

Overview

Design Code Family(s):	EN
Design Code(s):	EN 1992-1-1
Module(s):	AQB
Input file(s):	t-beam_shear.dat

1 Problem Description

The problem consists of a T-section, as shown in Fig. 1. The cross-section is designed for an ultimate shear force V_{Ed} and the required reinforcement is determined.

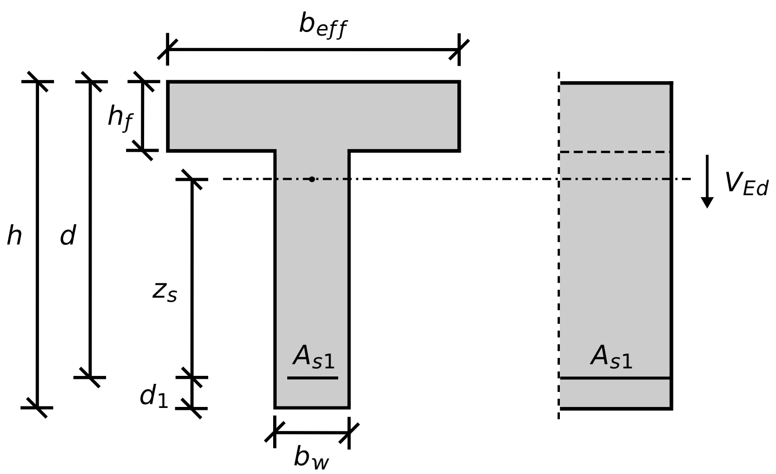

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force. The content of this problem is covered by the following parts of EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)
- Reinforcement (Section 9.2.2)

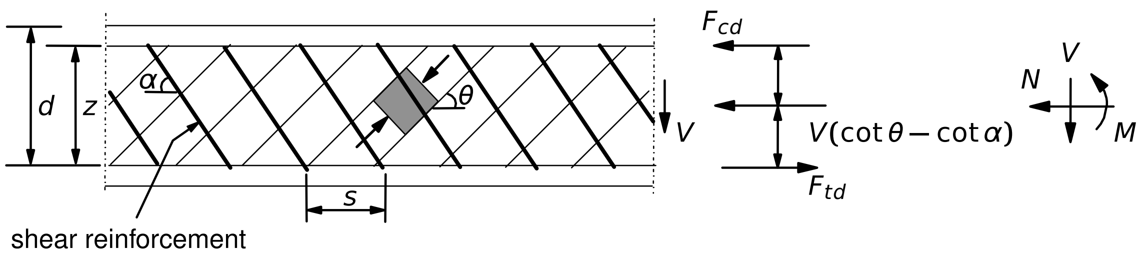

Figure 2: Shear Reinforced Members

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in EN 1992-1-1:2004 [1] (Section 3.2.7).

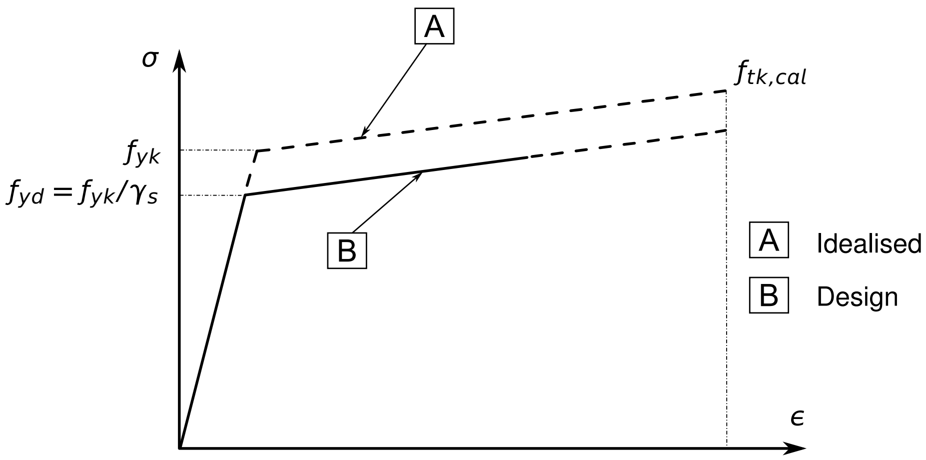

Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The T-section, with properties as defined in Table 1, is to be designed, with respect to EN 1992-1-1:2004 [1] to carry an ultimate shear force of $450\ kN$. The reference calculation steps are presented below and the results are given in Table 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\ 30/37$	$h = 60.0\ cm$	$V_{Ed} = 450\ kN$
$B\ 500A$	$d = 53.0\ cm$	
	$d_1 = 7.0\ cm$	
	$b = 30\ cm$	
	$b_{eff} = 180\ cm$	
	$h_f = 15\ cm$	
	$A_{s1} = 15\ cm^2$	

The intermediate steps of calculating the required reinforcement are also validated in this example. First we calculate the design value for the shear resistance $V_{Rd,c}$ for members not requiring shear reinforcement.

It gives a value of $V_{Rd,c} = 62.52\ kN$.

Checking the results in **AQB**, we can see that SOFiSTiK outputs also $V_{rd1,c} = 62.52\ kN$.

Just to test this result, if we input a shear force of $V_{Ed} = 62.51\ kN$ just below the value for $V_{Rd,c}$, **AQB** will not output any value for $\cot\theta$ and the minimum reinforcement will be printed (M). If we now give a value of $V_{Ed} = 62.53\ kN$ just larger than $V_{Rd,c}$, then **AQB** will start increasing $\cot\theta$ and the minimum reinforcement will be printed. If we continue increasing V_{Ed}, **AQB** will continue increasing $\cot\theta$ until it reaches the upper limit of$\cot\theta = 2.5$ with using the minimum reinforcement. If now the minimum reinforcement is exceeded, **AQB** starts calculating a value for the required reinforcement.

Another option to test this limit of $V_{Rd,c} = 62.52\ kN$, would be to keep $\cot\theta = 1.0$ and now with $V_{Ed} = 62.53\ kN$, **AQB** calculates a value for the required reinforcement larger than the minimum reinforcement. For the maximum value of the angle θ, hence $\cot\theta = 1.0$, the maximum value allowed for V_{Ed} can be calculated as $755.57\ kN$. This can be found in **AQB** results as the $V_{rd2,c} = 755.57\ kN$ for the case of $\cot\theta = 1.0$. Giving as an input a shear force just above this value $V_{Ed} = 755.58$ triggers a warning "Shear design not possible".

Next step is the validation of $V_{Rd,max}$. When the design shear force V_{Ed} exceeds $V_{Rd,max}$ then $\cot\theta$ must be decreased so that $V_{Ed} = V_{Rd,max}$. The reference result for $V_{Rd,max}$ is $521.08\ kN$. Inputing a value just below that, should give a $\cot\theta = 2.5$, whereas for a value just above should give $\cot\theta < 2.5$. This can be verified easily in **AQB** output for $V_{Ed} = 521.07$ and $521.09\ kN$, respectively.

Also the minimum reinforcement is calculated exactly by **AQB** with a value of $2.63\ cm^2/m$.

Table 2: Results

	SOF.	Ref.
$A_{sw,requ}\ /\ s\ \ [cm^2/m]$	8.68	8.679
$A_{sw,min}\ /\ s\ \ [cm^2/m]$	2.63	2.629
$V_{Rd,c}\ \ [kN]$	62.52	62.517
$V_{Rd,max}\ [kN]$	521.08	521.08
$V_{Ed,max}\ [kN]$	755.57	755.5

4 Design Process[1]

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

Material: Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

Tab. 3.1: Strength for concrete

$f_{ck} = 30\ MPa$

$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 1.0 \cdot 30/1.5 = 20.0\ MPa$

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$\alpha_{cc} = 1.0$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

3.2.7: (2), Fig. 3.8

$f_{yk} = 500\ MPa$

$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

Design Load: $V_{Ed} = 450.0\ kN$

Design with respect to EN 1992-1-1:2004 [1]:[2]

6.2.3 (1): Inner lever arm z

$z \approx 0.9 \cdot d = 0.9 \cdot 530 = 477\ mm$

6.2.2 (1): Design value for shear resistance $V_{Rd,c}$ for members not requiring design shear reinforcement

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

$$C_{Rd,c} = 0.18/\gamma_c = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1.6143 < 2.0$$

$$\rho_1 = \frac{A_{sl}}{b_w d} = 0.00 < 0.02$$

$$V_{Rd,c} = \left[0.12 \cdot 1.6143 \cdot (100 \cdot 0.0 \cdot 30)^{1/3} + 0\right] \cdot 0.3 \cdot 0.53$$

$$V_{Rd,c} = 0.00\ kN \geq V_{Rd,c,min}$$

$$V_{Rd,c,min} = (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$$

$$v_{min} = 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$$

$$v_{min} = 0.035 \cdot 1.6143 \cdot 30^{1/2} = 0.39319$$

$$V_{Rd,c,min} = (0.39319 + 0.0) \cdot 0.3 \cdot 0.53 = 0.062517\ MN$$

$$V_{Rd,c,min} = 62.517\ kN \rightarrow V_{Rd,c} = 62.517\ kN$$

$V_{Ed} > V_{Rd,c} \rightarrow$ shear reinforcement is required

6.2.3 (2): Eq. 6.7N: The angle θ should be limited by Eq. 6.7N

$1.0 \leq \cot\theta \leq 2.5 \rightarrow$ start with $\cot\theta = 2.50$

6.2.3 (3): Eq. 6.9

$$V_{Rd,max} = b_w \cdot z \cdot \nu \cdot f_{cd} / (\cot\theta + \tan\theta)$$

6.2.2 (6): Eq. 6.6N

$$\nu = 0.6 \cdot \left[1 - \frac{f_{ck}}{250}\right] = 0.528$$

Min. reinforcement:

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

[2] The sections mentioned in the margins refer to EN 1992-1-1:2004 [1], unless otherwise specified.

$$\rho_{w,min} = 0.08 \cdot \sqrt{f_{ck}}/f_{yk} = 0.08 \cdot \sqrt{30}/500 = 0.0008763$$ 9.2.2 (5): Eq. 9.5N

$$A_{sw,min}/s = \rho_{w,min} \cdot b_w \sin\alpha$$

$$A_{sw,min}/s = 0.0008763 \cdot 30 \cdot 100 = 2.629\ cm^2/m'$$

Required reinforcement:

$$A_{sw,requ}\ /\ s = \frac{V_{Ed}}{f_{ywd} \cdot z \cdot \cot\theta}$$ 6.2.3 (3): Eq. 6.8

- **For $V_{Ed} < V_{Rd,c}$**

Shear reinforcement not required (min. reinforcement). In this example min. reinforcement is disabled.

$$cot\theta = tan\theta = 1.0, \quad b_w = 0.3\ m, \quad z = 0.477\ m, \quad \nu_1 = 0.6$$

$$\alpha_w = 1.0$$

$$V_{Rd,max} = 1.0 \cdot 0.3 \cdot 0.477 \cdot 0.60 \cdot \frac{20}{1.0 + 1.0} = 0.85859\ MN$$

$$V_{Rd,max} = 858.59\ kN$$

- **For $V_{Ed} = 63.0\ kN > V_{Rd,c} = 62.57$:**

Calculating the $V_{Rd,max}$ value:

$$V_{Rd,max} = 0.3 \cdot 0.477 \cdot 0.528 \cdot \frac{20}{2.5 + 0.4} = 0.52108\ MN$$

$$V_{Rd,max} = 521.08\ kN \geq V_{Ed} = 63\ kN$$

Calculating the $A_{sw,requ}\ /\ s$ value:

$$A_{sw,requ}\ /\ s = \frac{0.063}{434.78 \cdot 0.477 \cdot 2.5} \cdot 100^2 = 1.2151\ cm^2$$

- **For $V_{Ed} = 63\ kN$ and $\cot\theta = 1.0$:**

$$V_{Ed} = 63\ kN > V_{Rd,c} = 62.51\ kN$$

Calculating the $V_{Rd,max}$ value:

$$V_{Rd,max} = 0.3 \cdot 0.477 \cdot 0.528 \cdot \frac{20}{1.0 + 1.0} = 0.7555\ MN$$

$$V_{Rd,max} = 755.5\ kN \geq V_{Ed} = 63\ kN$$

Calculating the $A_{sw,requ}\ /\ s$ value:

$$A_{sw,requ}\ /\ s = \frac{0.521}{434.78 \cdot 0.477 \cdot 1.0} \cdot 100^2 = 3.037\ cm^2$$

- **For $V_{Ed} = 756\ kN$ and $\cot\theta = 1.0$:**

$$V_{Ed} = 756\ kN > V_{Rd,c} = 62.51\ kN$$

$$V_{Ed} = 756\ kN > V_{Rd,max} = 755.5\ kN$$

Shear design not possible, because the $\cot\theta$ value is fixed and can't *be iterated.*

- **For $V_{Ed} = 521.10\ kN$:**

  $V_{Ed} = 521.10\ kN > V_{Rd,c} = 62.51\ kN$

  $V_{Ed} = 521.10\ kN > V_{Rd,max} = 521.08\ kN$

The $\cot\theta$ value is iterated until $V_{Rd,max} \geq V_{Ed}$

- **For $V_{Ed} = 450\ kN$:**

  $V_{Ed} = 450\ kN > V_{Rd,c} = 62.51\ kN$

  $V_{Ed} = 450\ kN < V_{Rd,max} = 521.08\ kN$

Calculating the $V_{Rd,max}$ value:

$$V_{Rd,max} = 0.3 \cdot 0.477 \cdot 0.528 \cdot \frac{20}{2.5 + 0.4} = 0.52108\ MN$$

$$V_{Rd,max} = 521.08\ kN \geq V_{Ed} = 450\ kN$$

Calculating the $A_{sw,requ}\ /\ s$ value:

$$A_{sw,requ}\ /\ s = \frac{0.450}{434.78 \cdot 0.477 \cdot 2.5} \cdot 100^2 = 8.679\ cm^2$$

5 Conclusion

This example shows the calculation of the required reinforcement for a T-beam under shear force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *EN 1992-1-1: Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2004.