Benchmark Example No. 54

Equivalent Linear Temperature Load

SOFiSTiK | 2024

VERiFiCATiON
BE54 Equivalent Linear Temperature Load

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Front Cover

6th Street Viaduct, Los Angeles Photo: Tobias Petschke

Overview	
Element Type(s):	
Analysis Type(s):	
Procedure(s):	
Topic(s):	Fire and temperature
Module(s):	SOFiLOAD
Input file(s):	eqv_linear_temp_load.dat

1 Problem Description

The following example is focused on verifying the effects of the nonlinear temperature gradient along the height of a beam's cross section. A simply supported beam (Figure 1a) is analyzed with the corresponding temperature distribution (heating and cooling profiles) in the cross section (Figure 1b). The internal stresses due to the nonlinear temperature gradient can be divided into stresses due to uniform and linear temperature component and into remaining self-equilibrating eigenstresses [1].

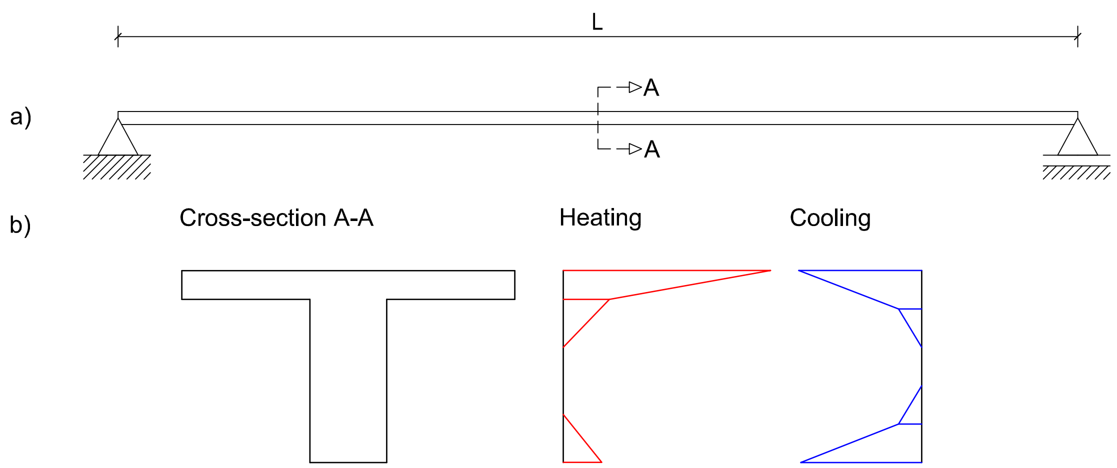


Figure 1: (a) Simply supported beam; (b) Cross section with corresponding heating and cooling profiles

2 Reference Solution

The reference solution is calculated analytically from the stress distribution corresponding to the restrained conditions, which is obtained by multiplying the assigned temperature profile with the coefficient of thermal expansion α_t and the modulus of elasticity E [2, 3] :

$$\sigma^T(z) = -E\alpha_t \Delta T(z), \tag{1}$$

Stress due to the restraining axial force is derived by integrating the stresses $\sigma^T(z)$ multiplied with the corresponding width $b(z)$ over the cross section height and dividing the value with the cross-section area A [2, 3]. The same stress value can be obtained by multiplying the equivalent uniform (constant) temperature component ΔT_{eq} with the coefficient of thermal expansion and the modulus of elasticity

(Figure 2b):

$$\sigma_{cons} = \frac{1}{A}\int_0^h \sigma^T(z)b(z)dz = -E\alpha_t \Delta T_{eq} \tag{2}$$

Stresses due to the restraining moment are calculated by taking moments around the centroid of the cross section and dividing the values with the section modulus [2, 3]. Correspondingly, the linear temperature distribution multiplied with the coefficient of thermal expansion and the modulus of elasticity (Figure 2c) yields the same stress values. Hence, the equivalent linear temperature component $\Delta T_{z,eq}$ can be derived from the following expression:

$$\sigma_{line} = \frac{1}{I/h}\int_0^h \sigma^T(z)b(z)(z-\bar{z})dz = -E\alpha_t \Delta T_{z,eq} \tag{3}$$

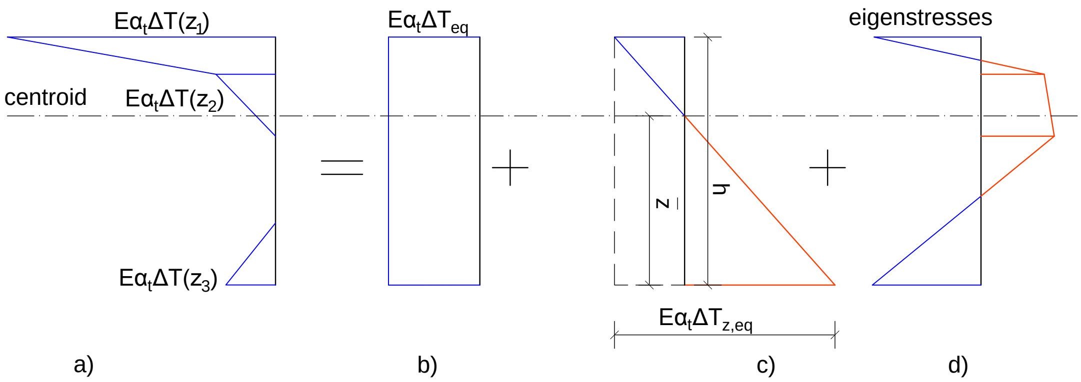


Figure 2: (a) Restrained stresses; (b) Stresses due to the equivalent uniform temperature; (c) Stresses due to the equivalent linear temperature; (d) Self-equilibrating eigenstresses

In case of a simply supported beam, it is free to expand and bend. Therefore, the corresponding strain distributions are generated. The differences between the restrained stress distribution and that which result in axial and bending strains, are trapped in the section and are known as self-equilibrating eigenstresses [3].

3 Model and Results

Two different cross-sections with the corresponding nonlinear temperature gradient are investigated: a concrete T-beam and a composite cross-section. The used material properties for concrete and steel are presented in Table 1. The implemented geometry and the temperature loading profiles for both heating and cooling conditions are shown in Figure 3. The beam's length is chosen to be 10 m. Reference solution for the same concrete T-beam cross-section, material properties and heating conditions can be found in [3].

Table 1: Material Properties

Type of cross section	Material properties	
T-beam	$E_{conc} = 35000\,MPa$	
	$\alpha_{t,conc} = 1.2 \times 10^{-5}\,K^{-}$	
Composite cross section	$E_{conc} = 35000\,MPa$	$E_{steel} = 210000\,MPa$
	$\alpha_{t,conc} = 1.2 \times 10^{-5}\,K^{-}$	$\alpha_{t,steel} = 1.2 \times 10^{-5}\,K^{-}$

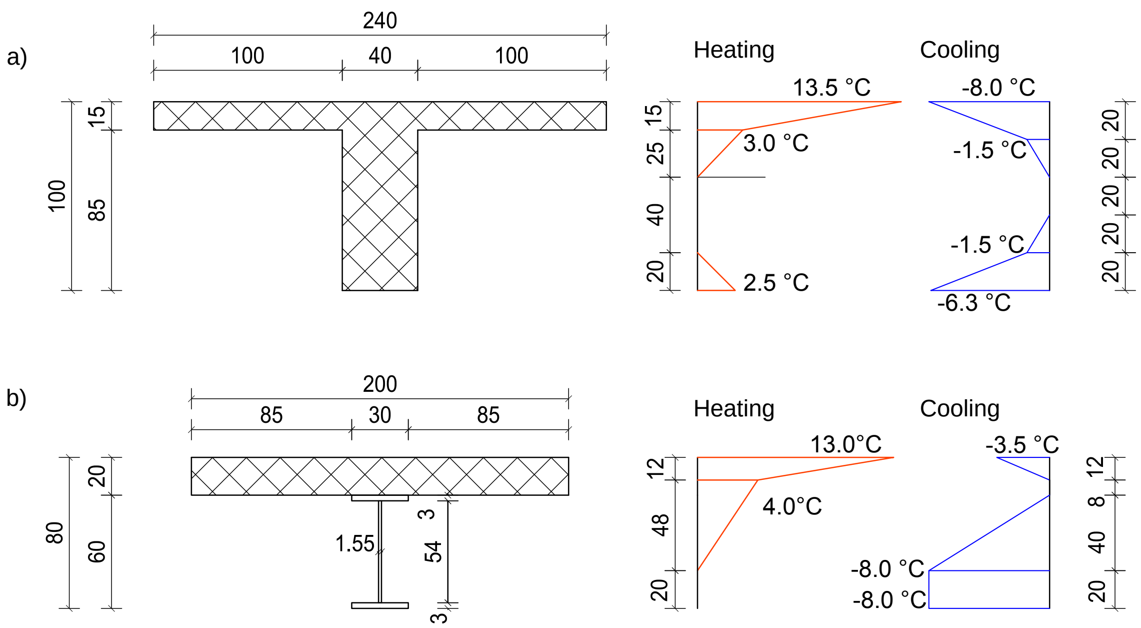


Figure 3: (a) T-beam concrete cross section - geometry [cm] with assigned temperature profiles; (b) Composite cross section - geometry [cm] with assigned temperature profiles

The calculated values of the equivalent uniform and linear temperature component are compared with the reference values in Table 2.

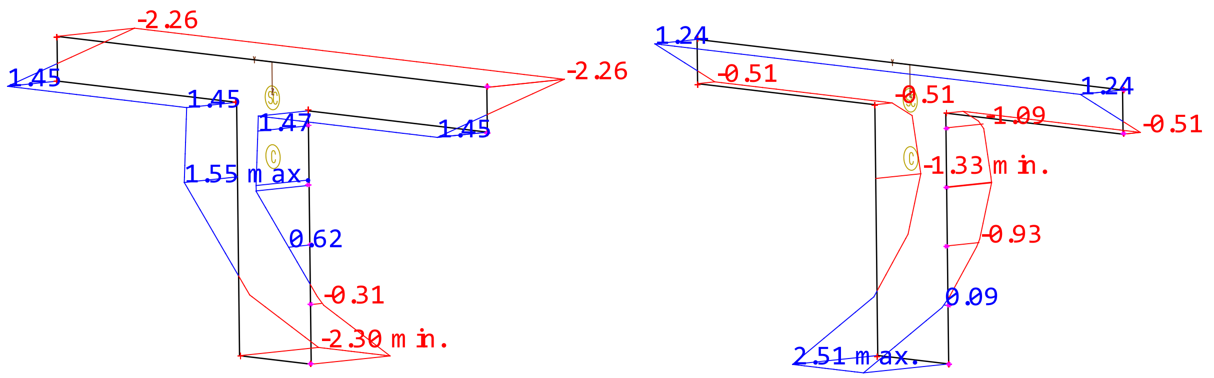
Table 2: Results

			ΔT_{eq} [$°C$]	$\Delta T_{z,eq}$ [$°C$]
T-beam	Heating	SOF.	4.600	-11.096
		Ref.[3]	4.600	-11.096
		$\lvert e_r \rvert$ [%]	0.00	0.00
	Cooling	SOF.	-3.544	4.704
		Ref.	-3.544	4.704
		$\lvert e_r \rvert$ [%]	0.00	0.00

Table 2: (continued)

			ΔT_{eq}	$\Delta T_{z,eq}$
			$[°C]$	$[°C]$
Composite	Heating	SOF.	5.111	−9.740
		Ref.	5.111	−9.740
		$\lvert e_r \rvert$ [%]	0.00	0.00
	Cooling	SOF.	−2.047	−7.371
		Ref.	−2.047	−7.371
		$\lvert e_r \rvert$ [%]	0.00	0.00

Calculated eigenstresses for a simply supported beam with the T-beam cross-section are shown in Figure 4. Results for the temperature heating profile calculated in [3] correspond nicely with the SOFiSTiK calculated eigenstresses (Figure 4a).



(a) For the heating profile (b) For the cooling profile

Figure 4: Eigenstresses for the T-beam cross-section in N/mm^2

4 Conclusion

An excellent agreement between the reference solution and the numerical results calculated by SOFiSTiK verifies that the effects of the nonlinear temperature gradient are adequately taken into account.

5 Literature

[1] M.M. Elbadry and A. Ghali. “Nonlinear temperature distribution and its effects on bridges”. In: *International Association of Bridge and Structural Engineering Proceedings* (1983), pp. 66–83.

[2] L.A. Clark. *Concrete Bridge Design to BS 5400*. Construction Press, 1983.

[3] D.L. Keogh and E. O´Brien. *Bridge Deck Analysis*. CRC Press, 2005.