Benchmark Example No. 2

Creep and Shrinkage Calculation using the Model Code 1990

SOFiSTiK | 2023

VERiFiCATiON
DCE-MC2 Creep and Shrinkage Calculation using the Model Code 1990

VERiFiCATiON Manual, Service Pack 2023-10 Build 44

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Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview	
Design Code Family(s):	MC
Design Code(s):	MC 1990
Module(s):	AQB, CSM
Input file(s):	creep_shrinkage_mc90.dat

1 Problem Description

The problem consists of a simply supported beam with a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The total creep and shrinkge is calculated.

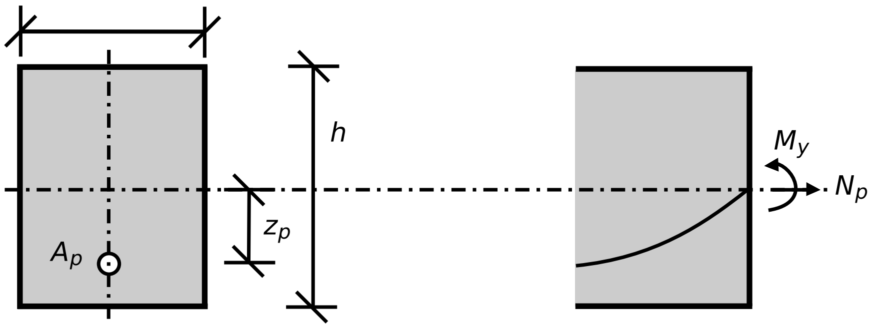

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the calculation of creep and shrinkage on a prestressed concrete cs, subject to bending and prestress force. The content of this problem is covered by the following parts of CEB-FIP Model Code 1990 [1]:

- Creep and Shrinkage (Section 2.1.6.4)
- Temperature effects (Section 2.1.8)

In this Benchmark the total creep and shrinkage will be examined.

3 Model and Results

Benchmark 17 is here extended for the case of creep and shrinkage developing on a prestressed concrete simply supported beam. The analysed system can be seen in Fig. 2, with properties as defined in Table 1. Further information about the tendon geometry and prestressing can be found in Benchmark 17. The beam consists of a rectangular cs and is prestressed and loaded with its own weight. A calculation of the creep and shrinkage is performed with respect to CEB-FIP Model Code 1990 [1].

Table 1: Model Properties

Material Properties	Geometric Properties	Time
$C\ 35/45$	$h = 100.0\ cm$	$t_0 = 28\ days$
$Y\ 1770$	$b = 100.0\ cm$	$t_s = 0\ days$
$RH = 80\ \%$	$L = 20.0\ m$	$t = 36500\ days$

Table 1: (continued)

Material Properties	Geometric Properties	Time
	$A_p = 28.5\ cm^2$	

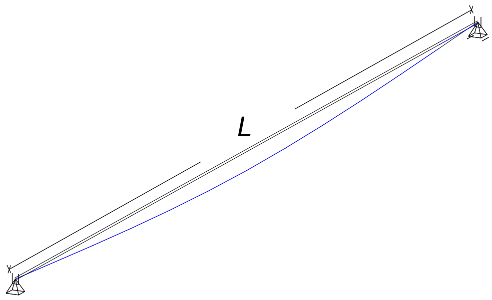

Figure 2: Simply Supported Beam

Table 2: Results

Result	AQB	CSM+AQB	Ref.
ϵ_{cs}	$-25.1 \cdot 10^{-5}$	$25.1 \cdot 10^{-5}$	$25.146 \cdot 10^{-5}$
ϕ_0	1.57	-	1.566
$\phi(t, t_0)$	1.48	1.476	1.47

Note: The results from SOFiSTiK are rounded for output.

4 Design Process

Design with respect to CEB-FIP Model Code 1990 [1]

Material:

Concrete: C 35/45

2.1 Concrete classification and constitutive relations

$E_{cm} = 35000\ N/mm^2$

2.1.4.2: Modulus of elasticity for C 35/45

$f_{ck} = 35\ N/mm^2$

$f_{cm} = 43\ N/mm^2$

2.1.3.2: Mean value of compressive strength f_{cm}. See the eq. (2.1-1)

Prestressing Steel: Y 1770

5.3: Prestressing Steel

$E_p = 195000\ N/mm^2$

E_p for wires

$f_{pk} = 1770\ N/mm^2$

f_{pk} Characteristic tensile strength of prestressing steel

CALCULATION OF TOTAL SHRINKAGE AND SWELLING at $x = 10.0\ m$ midspan:

$t_0 = 28$ days

t_0 age at first loading

$t_s = 0$ days

t_s concrete age at the beginning of shrinkage or swelling

$t = 36500$ days

t age of concrete at the moment considered

The total shrinkage or swelling strans $\epsilon_{cs}(t, t_s)$ may be calculated from
$\epsilon_{cs}(t, t_s) = \epsilon_{cs0} \cdot \beta_s(t - t_s)$

2.1.6.4.4: Eq. 2.1-74; $\epsilon_{cs}(t, t_s)$ is the total or swelling strain

Calculating the notional shrinkage:

The notional shrinkage coefficient may be obtained from

$\epsilon_{cs0} = \epsilon_s(f_{cm}) \cdot \beta_{RH}$

2.1.6.4.4: Eq. 2.1-75; ϵ_{cs0} is the notional shrinkage coefficient

with:

$$\epsilon_s(f_{cm}) = \left[160 + 10 \cdot \beta_{sc} \cdot \left(9 - \frac{f_{cm}}{f_{cm0}}\right)\right] \cdot 10^{-6}$$

2.1.6.4.4: Eq.2.1-76; β_{sc} is a coefficient which depends on the type of cement, for N class of cement $\beta_{sc} = 5$; $f_{cm0} = 10\ MPa$

$$\epsilon_s(f_{cm}) = \left[160 + 10 \cdot 5 \cdot \left(9 - \frac{43}{10}\right)\right] \cdot 10^{-6}$$

$$\epsilon_s(f_{cm}) = 39.5 \cdot 10^{-5}$$

$\beta_{RH} = -1.55 \cdot \beta_{sRH}$ for $40\ \% \leq RH < 99\ \%$

2.1.6.4.4: Eq.2.1-77

$$\beta_{sRH} = 1 - \left(\frac{RH}{RH_0}\right)^3 = 1 - \left(\frac{80}{100}\right)^3 = 0.488$$

2.1.6.4.4: Eq.2.1-78; $RH_0 = 100\ \%$

$$\beta_{RH} = -1.55 \cdot 0.488 = -0.7564$$

$$\epsilon_{cso} = 39.5 \cdot 10^{-5} \cdot (-0.7564) = -29.8778$$

2.1.6.4.4: Eq.2.1-79; $\beta_s(t-t_s)$ is the development of shrinkage with time; $h_0 = 100\ mm$; $t_1 = 1\ day$

The development of shrinkage with time is given by:

$$\beta_s(t-t_s) = \left[\frac{(t-t_s)/t_1}{350 \cdot (h/h_0)^2 + (t-t_s)/t_1}\right]^{0.5}$$

SOFiSTiK accounts not only for the age at start of drying t_s but also for the influence of the age of prestressing, so the time development function reads:

$$\beta_s = \beta_s(t-t_s) - \beta_s(t_0-t_s)$$

$$\beta_s = \left[\frac{36500}{350 \cdot 5^2 + 36500}\right]^{0.5} - \left[\frac{28}{350 \cdot 5^2 + 28}\right]^{0.5}$$

$$\beta_s = 0.8981 - 0.05647 = 0.8416$$

The total shrinkage or swelling strain is calculated:

$$\epsilon_{cs}(t,t_s) = \epsilon_{cs0} \cdot \beta_s$$

$$\epsilon_{cs}(t,t_s) = -29.8778 \cdot 10^{-5} \cdot 0.8416 = 25.146 \cdot 10^{-5}$$

CALCULATION OF TOTAL CREEP at x=10.0 m midspan:

2.1.6.4.3(b): Eq. 2.1-64; $\phi(t,t_0)$ is the creep coefficient

The creep coefficient may be calculated from:

$$\phi(t,t_0) = \phi_0 \cdot \beta_c(t-t_0)$$

2.1.6.4.3(b): Eq. 2.1-65; ϕ_0 is the notional creep coefficient

The notional creep coefficient may be estimated from:

$$\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

2.1.6.4.3(b): Eq.2.1-66; h is the notional size of member in [mm], $h = \frac{2 \cdot A_c}{u}$

with:

$$\phi_{RH} = 1 + \frac{1-(RH/RH_0)}{0.46 \cdot (h/h_0)^{1/3)}}$$

2.1.6.4.3(b): Eq. 2.1-66

$$\phi_{RH} = 1 + \frac{1-(80/100)}{0.46 \cdot (500/100)^{1/3)}} = 1 + \frac{0.2}{0.78658} = 1.254$$

2.1.6.4.3(b): Eq. 2.1-67

$$\beta(f_{cm}) = \frac{5.3}{(f_{cm}/f_{cm0})^{0.5}} = \frac{5.3}{(43/10)^{0.5}} = 2.556$$

2.1.8.2: Eq. 2.1-87; $t_{0,T}$ is the adjusted age of concrete at loading (days)

The adjusted time t_0 is given by:

$$t_{0,T} = \sum_{i=1}^{n} \Delta t_i \cdot exp\left[13.65 - \frac{4000}{273 + T(\Delta t_i)/T_0}\right]$$

$$t_{0,T} = \sum_{i=1}^{n} 28 \cdot exp\left[13.65 - \frac{4000}{273 + 20/1}\right] = 27.947 \text{ days}$$

2.1.6.4.3(c): Eq.2.1-71; the effect of type of cement on the creep coefficient of concrete may be taken into account by using the modified age at loading $t_{0,adj}$; $\alpha = 0$ for cement class N

$$t_{0,adj} = t_{0,T} \cdot \left[\frac{9}{2 + (t_{0,T}/t_{1,T}^{1.2})} + 1\right]^{\alpha} \geq 0.5\ days$$

$$t_{0,adj} = 27.947 \cdot \left[\frac{9}{2 + 27.947^{1.2}} + 1\right]^{0} = 27.947 \geq 0.5\ days$$

2.1.6.4.3(b): Eq. 2.1-68

$$\beta(t_0) = \frac{1}{0.1 + (t_0/t_1)^{0.2}} = \frac{1}{0.1 + (27.947/1)^{0.2}} = 0.48862$$

The development of creep with time is given by:

$$\beta_c(t-t_0) = \left[\frac{(t-t_0)/t_1}{\beta_h + (t-t_0)/t_1}\right]^{0.3}$$

2.1.6.4.3(b): Eq. 2.1-70

with:

$$\beta_H = 150 \cdot \left\{1 + \left(1.2 \cdot \frac{RH}{RH_0}\right)^{18}\right\} \cdot \frac{h}{h_0} + 250 \leq 1500$$

2.1.6.4.3(b): Eq. 2.1-71; $t_1 = 1\ day$; $RH_0 = 100\ \%$; $h_0 = 100\ mm$

$$\beta_H = 150 \cdot \left\{1 + \left(1.2 \cdot \frac{80}{100}\right)^{18}\right\} \cdot \frac{500}{100} + 250 \leq 1500$$

$$\beta_H = 1359.702 \leq 1500$$

$$\beta_c(t-t_0) = \left[\frac{(36500-28)/1}{1359.702 + (36500-28)/1}\right]^{0.3} = 0.989$$

$$\phi_0 = 1.254 \cdot 2.556 \cdot 0.48862 = 1.566$$

The creep coefficient:

$$\phi(t,t_0) = 1.56613 \cdot 0.989 = 1.5489$$

The creep value is related to the tangent Youngs modulus, where the tangent modulus being defined as $1.05 \cdot E_{cm}$. To account for this, SOFiSTiK adopts this scaling for the computed creep coefficient (in SOFiSTiK, all computations are consistently based on the secant modulus of elasticity).

$$\phi(t,t_0) = \frac{1.5489}{1.05} = 1.47$$

5 Conclusion

This example shows the calculation of the creep and shrinkage using Model Code 1990 [1]. It has been shown that the results are in very good agreement with the reference solution.

6 Literature

[1] CEB-FIP Model Code 1990. *Model Code for for Concrete Structures 1990*. Euro-International Concrete Committee. 1991.