Benchmark Example No. 25

Shear between web and flanges of Hollow CS acc. DIN EN 1992-2

SOFiSTiK | 2023

VERiFiCATiON
DCE-EN25 Shear between web and flanges of Hollow CS acc. DIN EN 1992-2

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Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1, DIN EN 1992-2
Module(s):	AQB
Input file(s):	hollow_shear_web_flange.dat

1 Problem Description

The problem consists of a Hollow section, as shown in Fig. 1. The cs is designed for shear, the shear between web and flanges of the Hollow CS is considered and the required reinforcement is determined.

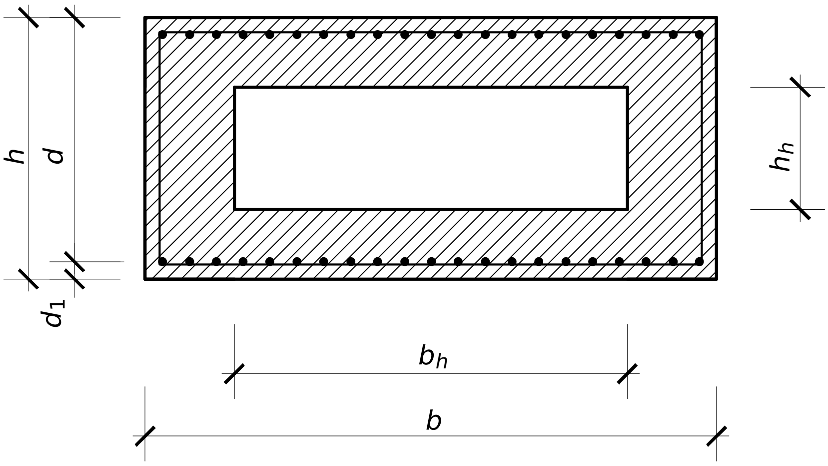

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the shear design of Hollow-sections, for the ultimate limit state. The content of this problem is covered by the following parts of DIN EN 1992-2:2010 [1] and DIN EN 1992-1-1:2004 [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 2 and as defined in DIN EN 1992-1-1:2004 [2] (Section 3.2.7).

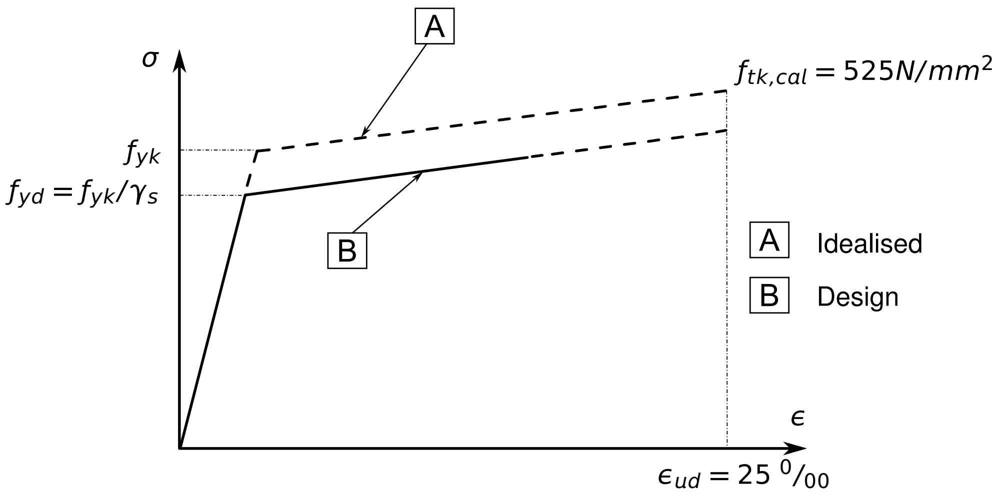

Figure 2: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The Hollow-section, with properties as defined in Table 1, is to be designed for shear, with respect to DIN EN 1992-2:2010 (German National Annex) [1]. The structure analysed, consists of a single span beam with a distributed load in gravity direction. The cross-section geometry, as well as the shear cut under consideration can be seen in Fig. 3.

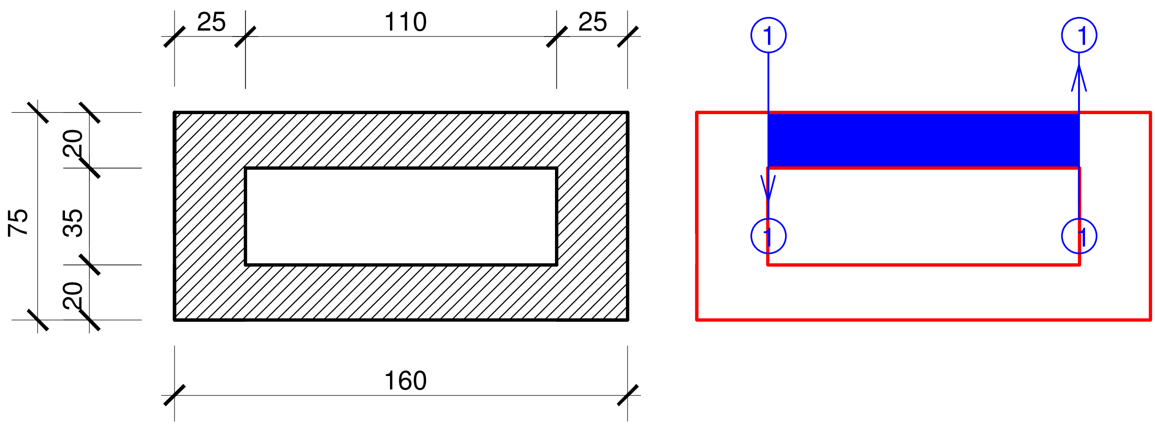

Figure 3: Cross-section Geometry, Properties and Shear Cuts in cm

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\ 30/37$	$h_h = 35\ cm, h = 75.0\ cm$	$P_g = 155\ kN/m$
$B\ 500B$	$h_f = 20\ cm$	$M_t = 100\ kN/m$
	$d_1 = 5.0\ cm$	
	$b_h = 110\ cm, b = 160\ cm$	

The system with its loading as well as the moment and shear force are shown in Fig. 4-7. The reference calculation steps are presented in the next section and the results are given in Table 2.

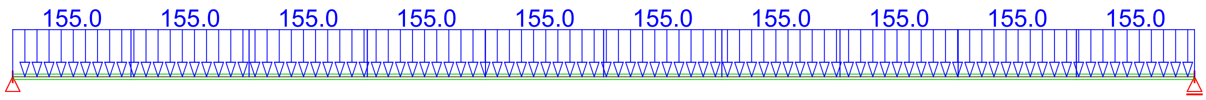

Figure 4: Loaded Structure (P_g)

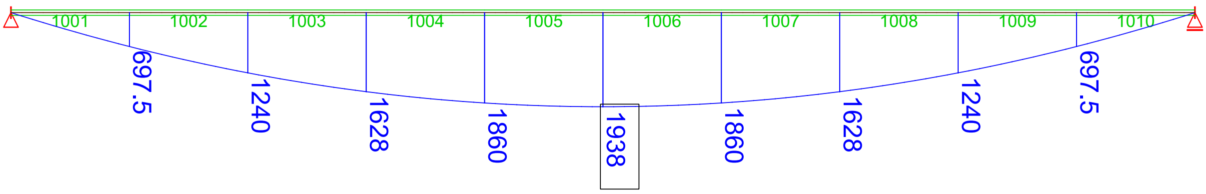

Figure 5: Resulting Bending Moment M_y

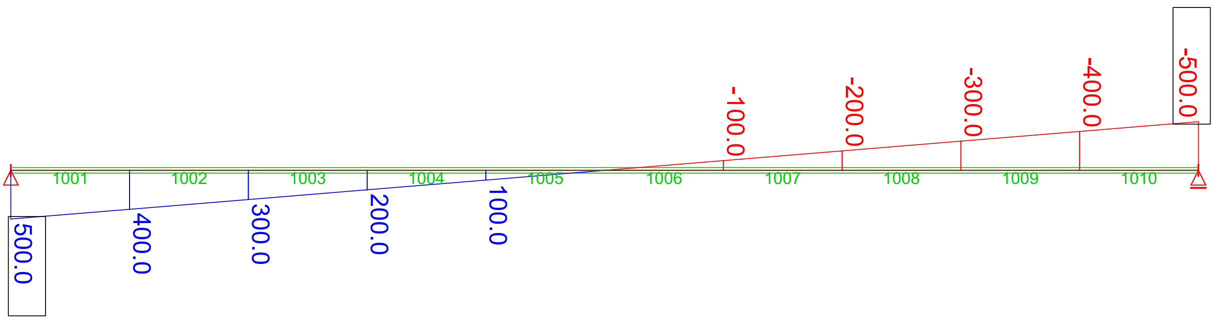

Figure 6: Resulting Torsional Moment M_t

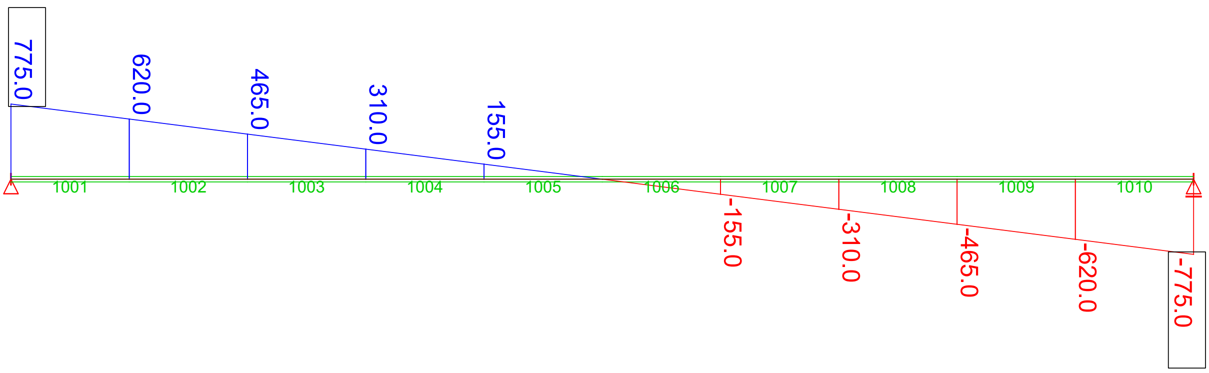

Figure 7: Resulting Shear Force V_z

Table 2: Results

At beam 1001	SOF.	Ref.
A_{s1} $[cm^2]$ at $x = 1.0\ m$	22.50	22.52
$V_{Rd,c}$ $[kN]$	91.91	91.91

Table 2: (continued)

At beam 1001	SOF.	Ref.
$V_{Rd,max}$ $[kN]$	1098.43	1098.461
$\cot\theta$	1.75	1.75
z $[cm]$ at $x = 1.0$ m	63.00	67.00
$V_{Ed} = \Delta F_d$ $[kN]$	346.64	352.35
$a_{sf,left}$ (x=0.00) $[cm^2]$	7.93	7.99
$a_{sf,right}$ (x=0.00) $[cm^2]$	1.19	1.27
$a_{sf,left}$ (x=1.00) $[cm^2]$	7.25	7.32
$a_{sf,right}$ (x=1.00) $[cm^2]$	1.86	1.94

4 Design Process[1]

Design with respect to DIN EN 1992-2:2010 (NA) [1] :[2]

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

$f_{ck} = 30\ MPa$
$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 30/1.5 = 17\ MPa$

Tab. 3.1: Strength for concrete

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$
$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

3.2.7: (2), Fig. 3.8

$\sigma_{sd} = 456.52\ MPa$

Design loads

- Design Load for beam 1001, x=0.0 m:

$M_{Ed,x=0.0\ m} = 0.0\ kNm$

- Design Load for beam 1001, x=1.0 m:

$M_{Ed,x=1.0\ m} = 697.5\ kNm$

Calculating the longitudinal reinforcement:

- For beam 1001, x=0.0 m

$$\mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{0.0 \cdot 10^{-3}}{1.60 \cdot 0.70^2 \cdot 17.00} = 0.00$$

$\mu = 0 \rightarrow A_{s1} = 0$

- For beam 1001, x=1.0 m

$$\mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{697.5 \cdot 10^{-3}}{1.60 \cdot 0.70^2 \cdot 17.00} = 0.0523$$

$\omega \approx 0.053973,\ \zeta \approx 0.9658,\ \xi = 0.07833$ (interpolated)

Tab. 9.2 [3]: ω—Table for up to $C50/60$ without compression reinforcement

$$A_{s1} = \frac{1}{\sigma_{sd}} \cdot (\omega \cdot b \cdot d \cdot f_{cd} + N_{Ed})$$

$N_{Ed} = 0$

$$A_{s1} = \frac{1}{456.52} \cdot (0.053973 \cdot 1.6 \cdot 0.7 \cdot 17.0) \cdot 100^2 = 22.52\ cm^2$$

$z = \zeta \cdot d = 0.9658 \cdot 0.70\ m \approx 67.00\ cm$

Calculating the shear between flange and web

The shear force, is determined by the change of the longitudinal force, at the junction between one side of a flange and the web, in the separated flange:

6.2.4 (3): Eq. 6.20

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [2] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 2.

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], unless otherwise specified.

$$x = 0.0738 \cdot d = 0.07833 \cdot 70 = 5.48 \text{ cm} < h_f = 20 \text{ cm}$$

$$\Delta F_d = \left(\frac{M_{Ed,x=1.0}}{z} - \frac{M_{Ed,x=0.0}}{z} \right) \cdot \frac{h_f \cdot b}{h_f \cdot b_h}$$

For beam 1001 (x=0.00 m) → $M_{Ed} = 0.00$ therefore:

In AQB output $\Delta F_d = T[kN/m]$

$$\Delta F_d = \left(\frac{697.5}{0.68047} - 0 \right) \cdot \frac{1.1}{1.6} = 352.35 \cdot 2 = 704.70\ kN$$

The longitudinal shear stress $v_{Ed,V}$ at the junction between one side of a flange and the web is determined by the change of the normal (longitudinal) force in the part of the flange considered, according to:

DIN EN 1992, 6.2.4 (3),Eq. 6.20: h_f is the thickness of flange at the junctions, Δx is the length under consideration, ΔF_d is the change of the normal force in the flange over the length Δx

$$v_{Ed,V} = \frac{\frac{\Delta Fd}{2}}{h_f \cdot \Delta x}$$

In our case $\Delta x = 1.0$ because the beam length is = 1.00 m.

Please note that AQB is outputting the results per length.

In AQB output $v_{Ed,V} = \tau - V$

$$v_{Ed,V} = \frac{352.35}{20 \cdot 100} = 0.176\ kN/m^2 = 1.76\ MPa$$

Checking the maximum $v_{Rd,max}$ value to prevent crushing of the struts in the flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied:

DIN EN 1992, 6.2.4 (4),Eq. 6.22

$$v_{Ed,V} \leq v_{Rd,max} = \nu \cdot f_{cd} \cdot \sin\theta_f \cdot \cos\theta_f$$

$$v_{Rd,max} = \nu \cdot f_{cd} \cdot \sin\theta_f \cdot \cos\theta_f$$

According to DIN EN 1992-2, NDP 6.2.4:

$$\nu = \nu_1$$

DIN EN 1992, NDP 6.2.3 (3),Eq. 6.22

$$\nu_1 = 0.75 \cdot \nu_2$$

$$\nu_2 = 1.1 - \frac{f_{ck}}{500} \leq 1.0$$

$$\nu_2 = 1.1 - \frac{30}{500} = 1.1 - 0.06 = 1.04 \geq 1.0 \rightarrow \nu_2 = 1.0$$

$$\nu_1 = 0.75 \cdot 1.0 = 0.75 \rightarrow \nu = 0.75$$

The θ value is calculated:

DIN EN 1992-2, NDP 6.2.3 (2),Eq. NA.6.bDE: $\sigma_{cd} = N_{Ed}/A_C$, c=0.5

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}}\right) \cdot b_w \cdot z$$

$$b_w \rightarrow h_f, \quad z \rightarrow \Delta x, \quad c = 0.5$$

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}}\right) \cdot h_f \cdot \Delta x$$

$$V_{Rd,cc\ (x=0.0\ m)} = 0.5 \cdot 0.48 \cdot 30^{1/3} \cdot \left(1 - 1.2 \cdot \frac{0}{17.00}\right) \cdot 0.20 \cdot 1.0$$

$$V_{Rd,cc\ (x=0.0\ m)} = 0.14914\ MN = 149.1471\ kN$$

$$V_{Rd,cc\ (x=1.0\ m)} = 0.5 \cdot 0.48 \cdot 30^{1/3} \cdot \left(1 - 1.2 \cdot \frac{1.069}{17.00}\right) \cdot 0.20 \cdot 1.0$$

$$V_{Rd,cc\ (x=1.0\ m)} = 0.137892\ MN = 137.892\ kN$$

$$1.0 \leq \cot\theta \leq \frac{1.2 + 1.4 \cdot \Delta\sigma_{cd}/f_{cd}}{1 - V_{Rd,cc\ (x=0.0\ m)}/V_{Ed}} \leq 1.75$$

$$1.0 \leq \cot\theta \leq \frac{1.2 + 1.4 \cdot \Delta\sigma_{cd}/f_{cd}}{1 - V_{Rd,cc\ (x=1.0\ m)}/V_{Ed}} \leq 1.75$$

DIN EN 1992-2, NDP 6.2.3 (2),Eq. NA.6.107aDE

Because $M_T \neq 0.0 \rightarrow \tau_T \neq 0.0$:

$$v_{Rd,cc\ (x=0.00)} = \frac{V_{Rd,cc}}{h_f \cdot \Delta x}$$

$$v_{Rd,cc\ (x=0.00)} = \frac{149.1471}{20 \cdot 100} = 0.0745\ kN/cm^2 = 0.745\ MPa$$

$$v_{Rd,cc\ (x=1.00)} = \frac{V_{Rd,cc}}{h_f \cdot \Delta x}$$

$$v_{Rd,cc\ (x=1.00)} = \frac{137.892}{20 \cdot 100} = 0.068946\ kN/cm^2 = 0.689\ MPa$$

The internal forces must be in equilibrium, see Fig. 8!!! If the internal forces are not in equilibrium state, then AQB will print erroneous results. Therefore the internal forces must be taken from the system (calculated by using e.g. ASE or STAR2).

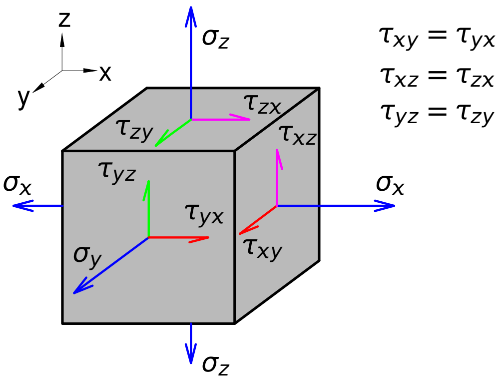

Figure 8: Stress equilibrium

$$\cot\theta_{(x=0.0)} = \frac{1.2}{1 - 0.745/1.76} = 2.0807 \geq 1.75 \rightarrow 1.75$$

$$\cot\theta_{(x=1.0)} = \frac{1.2 + 1.4 \cdot 1.069/17}{1 - 0.689/1.76} = 2.11 \geq 1.75 \rightarrow 1.75$$

AQB is checking and iterating if the $\cot\theta$ is less than (<) the maximum $cot\theta$ defined in the norm. If yes, then this value will be taken by AQB.

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{1.75} = 0.5714 \rightarrow \theta = 29.74488°$$

$$v_{Rd,max} = 0.75 \cdot 17.00 \cdot \sin 29.7448 \cdot \cos 29.7448 = 5.492\ MPa$$

$$V_{Rd,max} = v_{Rd,max} \cdot h_f \cdot \Delta x = 5.492 \cdot 0.20 \cdot 1.0 = 1.098\ MN$$

$$V_{Rd,max} = 1098.461\ kN$$

Checking the value $v_{Rd,c}$

If v_{Ed} is less than or equal to $v_{Rd,c} = k \cdot f_{ctd}$ no extra reinforcement above that for flexure is required.

$$v_{Rd,c} = k \cdot f_{ctd}$$

For concrete C 30/37 $\rightarrow f_{ctd} = 1.15\ MPa$

$$v_{Rd,c} = 0.4 \cdot 1.15 = 0.46\ MPa$$

$$V_{Rd,c} = v_{Rd,c} \cdot h_f \cdot \Delta x = 0.046 \cdot 20 \cdot 100 \approx 91.91\ kN$$

• Calculating the necessary transverse reinforcement (τ_V only):

DIN EN 1992, 6.2.4 (4),Eq. 6.21

$$a_{sf,V} = \frac{v_{Ed} \cdot h_f}{\cot \theta_f \cdot f_{yd}}$$

$$a_{sf,V} = \frac{1.76 \cdot 0.20}{1.75 \cdot 434.78} \cdot 100^2 = 4.63\ cm^2$$

• Calculating the necessary torsional reinforcement:

AQB is calculating the τ_t stresses by multiplying M_t with the torsional resistance $1/W_t$ (from AQUA - calculated by using FEM).

The program will set the upper limit to $M_t/(2 \cdot A_k \cdot t_{eff})$, if if one of the conditions is met:

- A_k is manually defined by the user.
  (AQUA input: SV → AK)
- The reinforcement contribution is set to torsional active.
  (AQUA will calculate and provide the A_k value to AQB)

$$\tau_{t\ (x=0.00)} = \frac{M_t}{2 \cdot A_k \cdot t_{eff}} = \frac{500 \cdot 10^{-3}}{2 \cdot 0.975 \cdot 0.2} = 1.282\ MPa$$

$$a_{sf,T\ (x=0.00)} = \frac{\tau_t \cdot t_{eff}}{\cot \theta_f \cdot f_{yd}} = \frac{1.282 \cdot 0.20}{1.75 \cdot 434.78} \cdot 100^2 = 3.36\ cm^2$$

$$\tau_{t\ (x=1.00)} = \frac{M_t}{2 \cdot A_k \cdot t_{eff}} = \frac{400 \cdot 10^{-3}}{2 \cdot 0.975 \cdot 0.2} = 1.025\ MPa$$

$$a_{sf,T\ (x=1.00)} = \frac{\tau_t \cdot t_{eff}}{\cot \theta_f \cdot f_{yd}} = \frac{1.025 \cdot 0.20}{1.75 \cdot 434.78} \cdot 100^2 = 2.69\ cm^2$$

• Total reinforcement:

$$a_{sf,left\ (x=0.00)} = a_{sf,V} + a_{sf,T} = 4.63 + 3.36 = 7.99\ cm^2$$

$$a_{sf,right\ (x=0.00)} = |a_{sf,V} - a_{sf,T}| = |4.63 - 3.36| = 1.27\ cm^2$$

$a_{sf,left\ (x=1.00)} = a_{sf,V} + a_{sf,T} = 4.63 + 2.69 = 7.32\ cm^2$

$a_{sf,right\ (x=1.00)} = |a_{sf,V} - a_{sf,T}| = |4.63 - 2.69| = 1.94\ cm^2$

5 Conclusion

This example is concerned with the calculation of the shear between web and flanges of a Hollow CS. It shows partially the work-flow how AQB calculates the shear between web and flanges.

Please note that it is very difficult to show all steps how AQB calculates internally the τ and σ stresses, therefore some steps are skipped. The reference example is just an approximation that shows the results by using hand-calculation. It has been shown that the results calculated by using hand-calculation and the AQB module are reproduced with very good accuracy.

The τ_t and τ_V values between cross-section points are interpolated and calculated by using Finite Element Method.

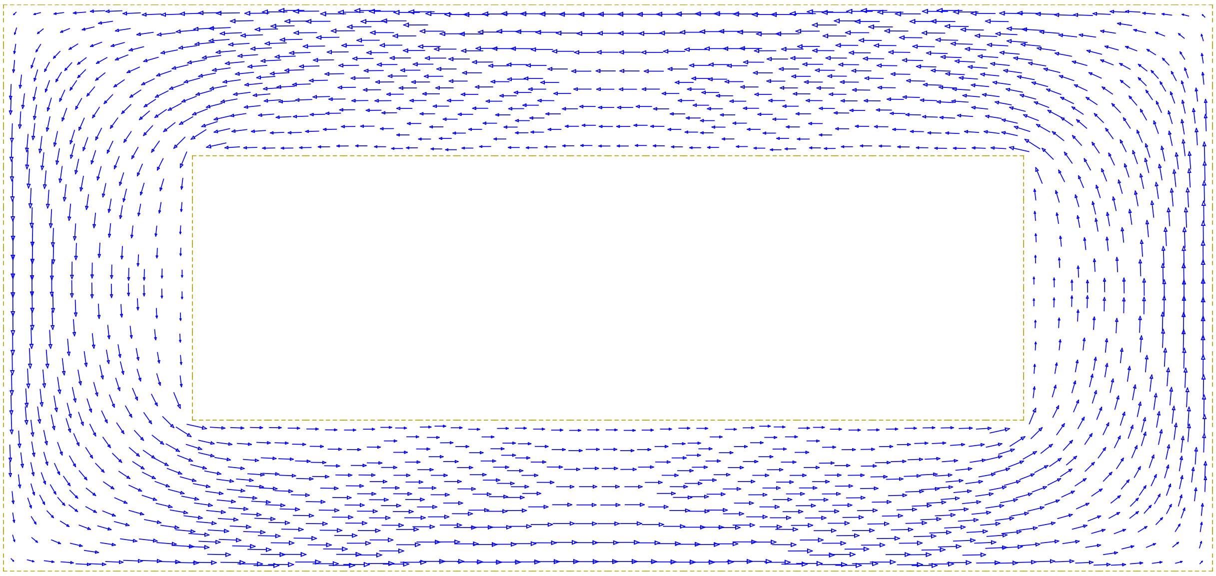

Figure 9: The τ_t stresses (vector) calculated by using Finite Element Method

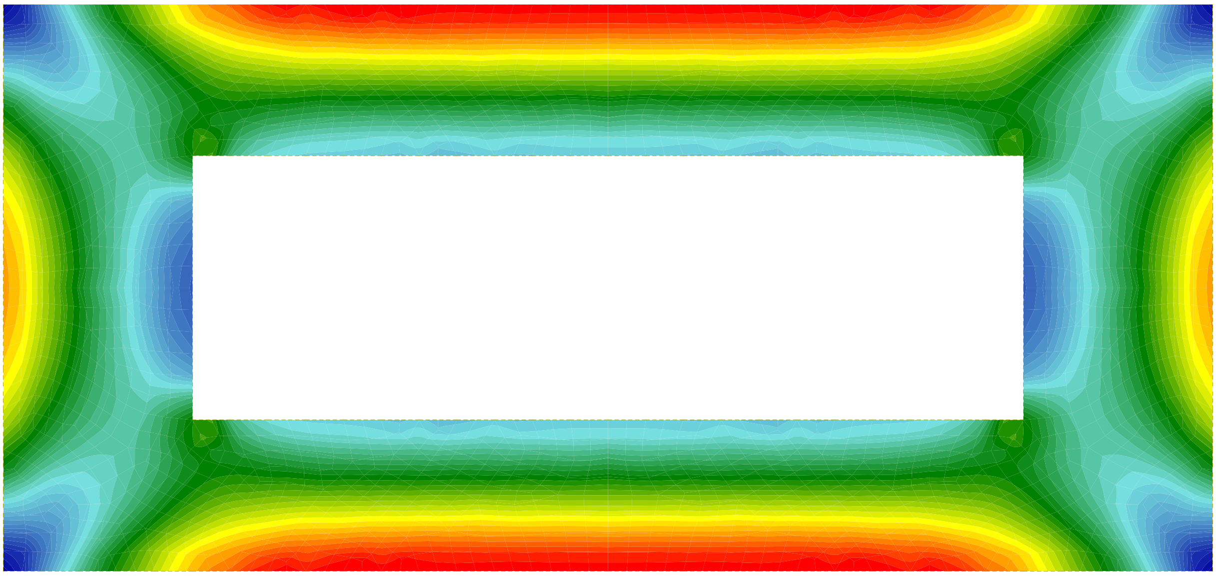

Figure 10: The τ_t stresses (fill) calculated by using Finite Element Method

6 Literature

[1] *DIN EN 1992-2: Eurocode 2: Design of concrete structures - Part 2, Part 2: Concrete Bridges - Design of Detailing Rules - German version EN 1992-2:2005 (D), Nationaler Anhang Deutschland - 2012*. CEN. 2012.

[2] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[3] K. Holschemacher, T. Müller, and F. Lobisch. *Bemessungshilfsmittel für Betonbauteile nach Eurocode 2 Bauingenieure*. 3rd. Ernst & Sohn, 2012.