Benchmark Example No. 20

Design of a Steel I-section for Bending, Shear and Axial Force

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VERiFiCATiON
DCE-EN20 Design of a Steel I-section for Bending, Shear and Axial Force

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Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview	
Design Code Family(s):	EN
Design Code(s):	EN 1993-1-1
Module(s):	AQB
Input file(s):	usage_interact.dat

1 Problem Description

The problem consists of a steel I section, as shown in Fig. 1. The cross-section is loaded with bending, shear and axial force. The required result is the utilization of the ultimate bearing capacity. The formulas provided in the Eurocode for that purpose are not generally applicable for all types of sections and do not allow a direct evaluation of this value, so it is common, that software implementations will deviate in some parts from the suggested procedure.

First it will be shown that a loading below the ultimate capacity will be covered by selected software even if deviations in the applied formulas will affect the expected results slightly. Then it will be demonstrated that an advanced analysis using additional features within the provisions of the design code allows a better approach to the ultimate capacity.

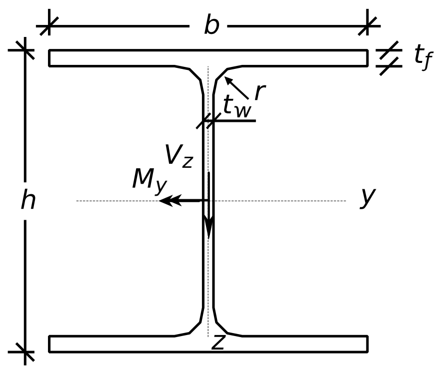

Figure 1: Problem Description

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$S\ 235$	HEM 500	$V_z = 1400\ kN$
$\gamma_{M0} = 1.00$	$b = 306\ mm$	$M_y = 450\ kNm$
	$h = 524\ mm$	$N = -5000\ kN$
	$t_f = 40\ mm$, $t_w = 21\ mm$	
	$r = 27\ mm$	
	$A = 344.298\ cm^2$	Sectional Class 1

2 Reference Solution

As significant shear and axial forces are present allowance should be made for the effect of both shear force and axial force on the resistance moment. The content of this problem is covered by the following parts of EN 1993-1-1:2005 [1]:

- Structural steel (Section 3.2)
- Resistance of cross-sections (Section 6.2)

3 Model and Results

The section, a HEM 500, with properties as defined in Table 1, is to be designed for an ultimate moment M_y, a shear force V_z and an axial force N, with respect to EN 1993-1-1:2005 [1]. The utilisation level of allowable plastic forces are calculated and compared to software results. The results are given in Table 2.

Table 2: Results

Result	SOF.	Ref.
Util. level N	0.618	0.618
Util. level $N_{Ed}/N_{V,Rd}$	0.712	0.712
Util. level V_z	0.797	0.797
Util. level M_y	0.270	0.270
Total util. level $M_{y,red}$ (EN 1993-1-1)	0.952	0.846

4 Design Process[1]

The calculation steps of the reference solution are presented below. Neither the axial force (see `EN 1993-1-1, 6.2.9.1 (4)`) nor the shear force (see `EN 1993-1-1, 6.2.10`) can be neglected.

Material:

Structural Steel S 235

$f_y = 235\ N/mm^2$

Cross-sectional properties:

$W_{pl,y} = 7094.2\ cm^2$

Plastic section modulus of HEM 500 w.r.t. y-

$A_{V,z} = A - 2 \cdot b \cdot t_f + (t_w + 2 \cdot r) \cdot t_f$ but not less than $\eta \cdot h_w \cdot t_w$

6.2.6 (3)a: The shear area $A_{V,z}$ may be taken as follows for rolled I-sections with load parallel to the web

$h_w = h - 2 \cdot t_f = 44.4\ cm$

η may be conservatively taken equal to 1.0

$A_{V,z} = 129.498\ cm^2 > 1 \cdot 44.4 \cdot 2.1 = 93.24$

Partial factor $\gamma_{M0} = 1.0$

$$\alpha_{V,z} = \frac{A_{V,z}}{A} = \frac{129.5}{344.3} = 0.376$$

$A_v = A - 2 \cdot b \cdot t_f = 99.498\ cm^2$

$$a = \frac{A_V}{A} = \frac{99.5}{344.3} = 0.289$$

6.2.9.1 (5): $a = \frac{A - 2 \cdot b \cdot t_f}{A}$

$$V_{pl,Rd,z} = \frac{129.498 \cdot 235/\sqrt{3}}{1.00} = 1757\ kN$$

6.2.6 (3)a: The shear area A_{Vz} may be taken as follows for rolled I - sections with load parallel to the web

$$M_{c,Rd} = \frac{W_{pl,y} \cdot f_y}{\gamma_{M0}} = 1667\ kNm$$

6.2.5 (2), Eq. 6.13: The design resistance for bending, for Class 1 cross-section

Where the shear and axial force are present allowance should be made for the effect of both shear force and axial force on resistance moment.

6.2.10 (1): Bending, shear and axial force

Provided that the design value of the shear force V_{Ed} does not exceed the 50% of the design plastic shear resistance $V_{pl,Rd}$ no reduction of the resistances for bending and axial force need to be made.

$V_{Ed} \leq 0.5 \cdot V_{pl,Rd}$

6.2.10 (2)

$$\frac{V_{Ed}}{V_{pl,Rd,z}} = 0.797 > 0.5$$

→ shear resistance limit exceeded

6.2.10 (3): Bending shear and axial force

Provided that the design value of the shear force V_{Ed} exceeds the 50% of the design plastic shear resistance $V_{pl,Rd,z}$, the design resistance of the cross-section to combinations of moment and axial force should be calculated using a reduced yield strength $(1 - \rho) \cdot f_y$ for the shear area. Therefore all following formulas will be modified for the shear force allowance.

[1] The sections mentioned in the margins refer to EN 1993-1-1:2005 [1] unless otherwise specified.

6.2.10 (3) the design resistance should be calculated using a reduced yield strength $(1-\rho)\cdot f_y$ for the **shear area**

$$\rho = \left(\frac{2 \cdot V_{Ed}}{V_{pl,Rd,z}} - 1\right)^2 = 0.352$$

6.2.8 (5): Eq. 6.30: The reduced design plastic resistance moment allowing for shear force may alternatively be obtained for I-sections with equal flanges and bending about major axis

$$M_{y,V,Rd} = \frac{\left[W_{pl,y} - \dfrac{\rho \cdot A_w^2}{4 \cdot t_w}\right] \cdot f_y}{\gamma_{M0}} = 1581\ kNm$$

$$\frac{M_y}{M_{y,V,Rd}} = 0.277$$

$$\frac{M_y}{M_{c,Rd}} = 0.270 \text{ with no reduction}$$

6.2.4 (2), Eq. 6.10: The design resistance for compression

$$N_{pl,Rd} = \frac{A \cdot f_y}{\gamma_{M0}} = 8091\ kN$$

$$N_{V,Rd} = N_{pl,Rd} \cdot (1 - \alpha_{V,z} \cdot \rho)$$

$$N_{V,Rd} = 7018.6\ kN$$

$$n_V = \frac{N_{Ed}}{N_{V,Rd}} = 0.712$$

$$\frac{N_{Ed}}{N_{pl,Rd}} = 0.618 \text{ with no reduction}$$

6.2.9.1 (1): Bending and axial force - Class 1 and 2 cross-sections

Where an axial force is present, allowance should be made for its effect on the plastic moment resistance.

6.2.9.1 (4): For doubly symmetrical I-sections, allowance need not to be made for the effect of the axial force on the plastic resistance moment about the y-y axis when the criteria are satisfied.

$$N_{Ed} > min \begin{cases} 0.25 \cdot N_{V,Rd} \\ \dfrac{0.5 \cdot h_w \cdot t_w \cdot f_y \cdot (1-\rho)}{\gamma_{M0}} \end{cases}$$

$$N_{Ed} = 5000 > min \begin{cases} 1754.7\ kN \\ 709.5\ kN \end{cases}$$

$\rightarrow$ consideration of axial force in interaction

6.2.9.1 (5): Eq. 6.36-6.38: reduced design plastic resistance moment allowing for I-sections with equal flanges and bending about major axis.

$$M_{N,y,Rd} = M_{V,y,Rd} \cdot \frac{1-n}{1-0.5 \cdot \alpha*}$$

and $M_{N,y,Rd} \leq M_{V,y,Rd}$

$$n = \frac{N_{Ed}}{N_{V,Rd}} = 0.712$$

$$\alpha* = min \begin{cases} \dfrac{A - 2 \cdot b \cdot t_f}{A} = \dfrac{344.4 - 2 \cdot 30.6 \cdot 4}{344.4} = 0.289 \\ 0.5 \end{cases}$$

$$\alpha* = 0.289$$

$$M_{N,y,Rd} = 1581 \cdot \frac{1-0.712}{1-0.5 \cdot 0.289} = 531\ kNm$$

$$\frac{M_{y,Ed}}{M_{NV,y,Rd}} = \frac{450}{531} = 0.846$$

Alternative solution (see evaluation below):

$$M_{y,V,Rd} = \left(7094 - \frac{0.352 \cdot 129.5^2}{4 \cdot 2.1}\right) \cdot f_y$$

$$M_{y,V,Rd} = 1501.8\ kNm$$

$$M_{N,y,Rd} = \frac{1 - 0.712}{1 - 0.5 \cdot 0.376} \cdot 1501 = 532\ kNm$$

$$\frac{M_{Ed}}{M_{NV,y,Rd}} = \frac{450}{532} = 0.846$$

Using $A_w = A_v = A_{Vz}$ insead of three different values: $93.2 \neq 99.5 \neq 129.5$
Hit: The reduction by the plastic bending resistance of the web may be obtained with a better precision.

5 Conclusion

Interaction formulas are not expected to be true for all cases [2]. Thus deviations are unavoidable.

The first reason of the deviation is that the interaction formulas are not linear. Thus an utilisation factor of 0.5 does not mean that the ultimate load is twice the current loading. If the utilisation factor of the normal force or the shear force become 1.0 or larger then the formulas are not applicable any more. So the utilisation formula should be rewritten from the original form of:

$$\frac{M}{M_{y,V,Rd}} \cdot \frac{1-0.5\cdot a}{\left(1-\frac{N}{N_{V,Rd}}\right)} \leq 1.00 \tag{1}$$

to the completely equivalent form:

$$\frac{M}{M_{y,V,Rd}} \cdot (1-0.5\cdot a_{Vz}) + \frac{N}{N_{V,rd}} \leq 1.00 \tag{2}$$

These are two different curves intersecting for the critical value of 1.0, but the singularity for the utilisation $N/N_{V,Rd} = 1$ is avoided. The second formula yields a value of 0.956 with the EN reference data. The simplified interaction according to equation 6.2 of EN 1993-1-1 yields an utilisation value of 1.04. Program RUBSTAHL QST-I [2] yields an utilisation of 1.00.

The second reason for the deviations is that the software follows a more general principle and uses only a single value for the shear capacity as indicated in the last row of table 2. Thus the software has to select either the web only or the dog-bone-shape including parts of the flanges, as can be visualized in Fig. 2.

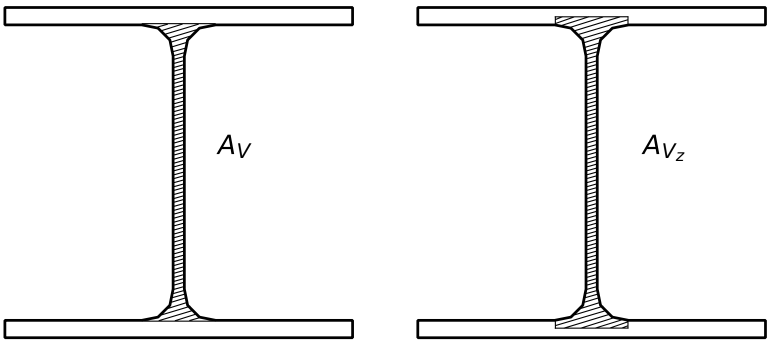

Figure 2: Different shear areas for steel section

These deviations become larger, if we try to find the true capacity of the section. It has to be noted, that most interaction formulas for shear and normal stress are working with partial areas and are not accounting for the true interaction of stresses. There are different possibilities in the literature and there are some numerical procedures by Katz [3] or Osterrieder [4] allowing for more detailed evaluations.

The solution with the optimization process from Osterrieder [4] yields a true utilisation factor of 0.93 for the given forces. Thus the true ultimate plastic force combination is: $N = 5380\ kN$, $V_z = 1505\ kN$, $M_y = 483.9\ kNm$. The software provides for these forces with the modified interaction formulas an utilisation of 1.08. If one follows strictly the design code, much higher values of about 2.7 are obtained, showing the strong non proportionality of those formulas.

The solution according to Katz [3] uses a non-linear analysis, thus we have finite strains and the hardening effect of the steel strain law is used, as indicated in EN 1993-1-5:2006 Appendix C.6 [5]. Then it is no problem to obtain even higher ultimate forces: $N = 5800\ kN$, $V_z = 1624\ kN$, $M_y = 522\ kNm$, giving a utilisation factor of 0.86 for the given loading. The normal and shear stress distribution in Figure 3 is obtained with a maximum compressive strain of $-1.61‰$.

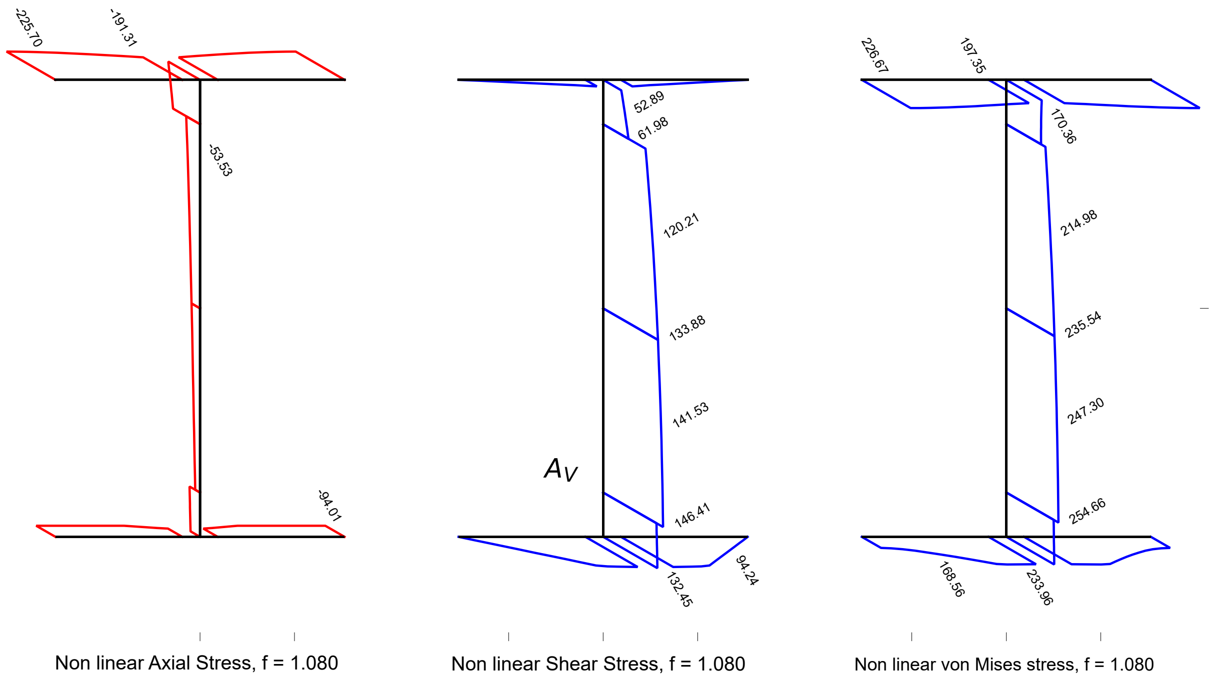

Figure 3: Non-linear normal and shear stress distribution

6 Literature

[1] *EN 1993-1-1: Eurocode 3: Design of steel structures, Part 1-1: General rules and rules for buildings*. CEN. 2005.

[2] R. Kindmann and U. Krüger. *Stahlbau, Teil 1: Grundlagen*. Ernst & Sohn, 2015.

[3] C. Katz. “Fließzonentheorie mit Interaktion aller Stabschnittgrößen bei Stahltragwerken”. In: *Stahlbau* (1997).

[4] P. Osterrieder. “Plastic Bending and Torsion of Open Thin-Walled Steel Members”. In: *Proceedings of EUROSTEEL 2005, 4th European Conference on Steel and Composite Structures* (Maastricht, 2005).

[5] *DIN EN 1993-1-5: Eurocode 3: Design of steel structures, Part 1-5: Plated structural elements - German version EN 1993-1-1:2006 + AC:2009*. CEN. 2010.