SOFiSTiK

Benchmark Example No. 18

Creep and Shrinkage Calculation of a Rectangular Prestressed Concrete CS

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VERiFiCATiON
DCE-EN18 Creep and Shrinkage Calculation of a Rectangular Prestressed Concrete CS

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SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB, CSM
Input file(s):	creep_shrinkage.dat

1 Problem Description

The problem consists of a simply supported beam with a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The time dependent losses are calculated, considering the reduction of stress caused by the deformation of concrete due to creep and shrinkage, under the permanent loads.

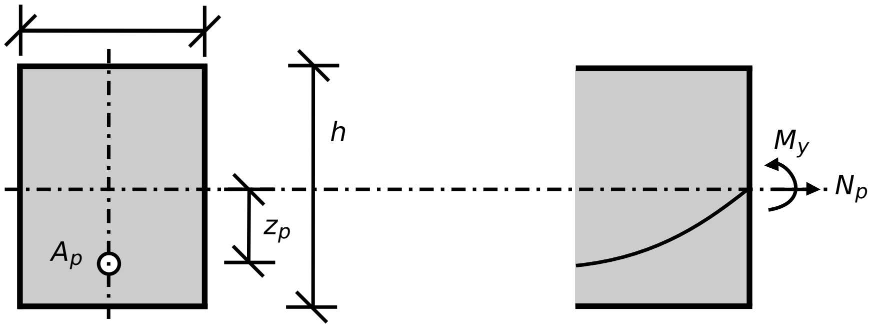


Figure 1: Problem Description

2 Reference Solution

This example is concerned with the calculation of creep and shrinkage on a prestressed concrete cs, subject to bending and prestress force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Creep and Shrinkage (Section $3.1.4$)
- Annex B: Creep and Shrinkage (Section $B.1$, $B.2$)
- Time dependent losses of prestress for pre- and post-tensioning (Section $5.10.6$)

The time dependant losses may be calculated by considering the following two reductions of stress [1]:

- due to the reduction of strain, caused by the deformation of concrete due to creep and shrinkage, under the permanent loads
- the reduction of stress in the steel due to the relaxation under tension.

In this Benchmark the stress loss due to creep and shrinkage will be examined.

3 Model and Results

Benchmark 17 is here extended for the case of creep and shrinkage developing on a prestressed concrete simply supported beam. The analysed system can be seen in Fig. 2, with properties as defined in Table 1. Further information about the tendon geometry and prestressing can be found in Benchmark 17. The beam consists of a rectangular cs and is prestressed and loaded with its own weight. A calculation of the creep and shrinkage is performed in the middle of the span with respect to DIN EN 1992-1-1:2004

(German National Annex) [1], [2]. The calculation steps [3] are presented below and the results are given in Table 2 for the calculation with AQB. For CSM only the results of the creep coefficients and the final losses are given, since the calculation is performed in steps.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading (at $x = 10\ m$)	Time
$C\ 35/45$	$h = 100.0\ cm$	$M_g = 1250\ kNm$	$t_0 = 28\ days$
$Y\ 1770$	$b = 100.0\ cm$	$N_p = -3653.0\ kN$	$t_s = 0\ days$
$RH = 80$	$L = 20.0\ m$		$t_{eff} = 1000000\ days$
	$A_p = 28.5\ cm^2$		

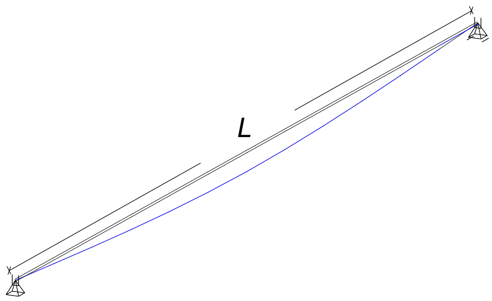


Figure 2: Simply Supported Beam

Table 2: Results

Result	AQB	CSM+AQB	Ref.
ϵ_{cs}	$-18.85 \cdot 10^{-5}$	-	$-18.85 \cdot 10^{-5}$
ϵ	$-31.58 \cdot 10^{-5}$	-	$-31.58 \cdot 10^{-5}$
ϕ_0	1.463	1.463	1.463
$\phi(t, t_0)$	1.393	1.393	1.393
$\Delta\sigma_{p,c+s}\ \ [MPa]$	-66.62	-67.30	-68.45
$\Delta P_{c+s}\ \ [kN]$	189.9	191.8	195.11

4 Design Process[1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

Concrete: $C\ 35/45$ — 3.1: Concrete

$E_{cm} = 34077\ N/mm^2$ — 3.1.2: Tab. 3.1: E_{cm}, f_{ck} and f_{cm} for $C\ 35/45$

$f_{ck} = 35\ N/mm^2$

$f_{cm} = 43\ N/mm^2$

Prestressing Steel: $Y\ 1770$ — 3.3: Prestressing Steel

$E_p = 195000\ N/mm^2$ — 3.3.6 (3): E_p for wires

$f_{pk} = 1770\ N/mm^2$ — 3.3.2, 3.3.3: f_{pk} Characteristic tensile strength of prestressing steel

Prestressing system: $BBV\ L19\ 150\ mm^2$

19 wires with area of $150\ mm^2$ each, giving a total of $A_p = 28.5\ cm^2$

Cross-section:

$A_c = 1.0 \cdot 1.0 = 1\ m^2$

Diameter of duct $\phi_{duct} = 97\ mm$

Ratio $\alpha_{E,p} = E_p\ /\ E_{cm} = 195000\ /\ 34077 = 5.7223$

$A_{c,netto} = A_c - \pi \cdot (\phi_{duct}/2)^2 = 0.9926\ m^2$

$A_{ideal} = A_c + A_p \cdot \alpha_{E,p} = 1.013\ m^2$

Load Actions:

Self weight per length: $\gamma = 25\ kN/m$

At $x = 10.0\ m$ middle of the span:

$M_g = g_1 \cdot L^2\ /\ 8 = 1250\ kNm$

$N_p = P_{m0}(x = 10.0\ m) = -3653.0\ kN$ (from SOFiSTiK)

Calculation of stresses at $x = 10.0\ m$ midspan:

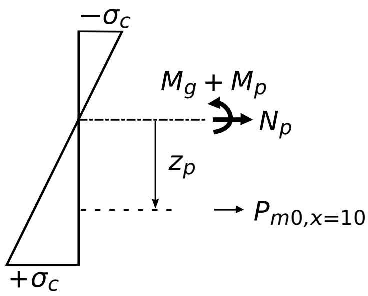
Position of the tendon: $z_{cp} = 0,3901\ m$

Prestress and self-weight at con. stage sect. 0 (P+G cs0)

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.3.6: Fig. 3.10

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

$N_p = -3653.0\ kN$ and $M_g = 1250\ kNm$



z_s the new position of the center of the cross-section for cs0
$z_p = z_{cp} + z_s$

M_p bending moment caused by prestressing

$$M_{p_1} = N_P \cdot z_{cp} = -3653.0 \cdot 0.3901 = -1425.04\ kNm$$

$$M_{p_2} = N_P \cdot z_s = -3653.0 \cdot 0.002978 = -10.879\ kNm$$

$$M_p = -1425.04 - 10.879 = -1435.91\ kNm$$

$$M_y = 1250 - 1435.91 = -185.91\ kNm$$

$\sigma_{c,QP}$ stress in concrete

$$\sigma_{c,QP} = \frac{-3653.0}{0.9926} + \frac{-185.91}{0.1633} = -4.82\ MPa$$

Calculation of creep and shrinkage at $x = 10.0\ m$ midspan:

t_0 minimun age of concrete for loading
t_s age of concrete at start of drying shrinkage
t age of concrete at the moment considered

$t_0 = 28$ days

$t_s = 0$ days

$t = t_{eff} + t_0 = 1000000 + 28 = 1000028$ days

3.1.4 (6): Eq. 3.8: ϵ_{cs} total shrinkage strain

$$\epsilon_{cs} = \epsilon_{cd} + \epsilon_{ca}$$

3.1.4 (6): Eq. 3.9: ϵ_{cd} drying shrinkage strain

$$\epsilon_{cd}(t) = \beta_{ds}(t, ts) \cdot k_h \cdot \epsilon_{cd,0}$$

The development of the drying shrinkage strain in time is strongly depends on $\beta_{ds}(t, ts)$ factor. SOFiSTiK accounts not only for the age at start of drying t_s but also for the influence of the age of the prestressing t_0. Therefore, the calculation of factor β_{ds} reads:

3.1.4 (6): Eq. 3.10: β_{ds}

$$\beta_{ds} = \beta_{ds}(t, t_s) - \beta_{ds}(t_0, t_s)$$

3.1.4 (6): h_0 the notional size (mm) of the cs $h_0 = 2A_c/u = 500\ mm$

$$\beta_{ds} = \frac{(t - t_s)}{(t - t_s) + 0.04 \cdot \sqrt{h_0^3}} - \frac{(t_0 - t_s)}{(t_0 - t_s) + 0.04 \cdot \sqrt{h_0^3}}$$

$$\beta_{ds} = \frac{(1000028 - 0)}{(1000028 - 0) + 0.04 \cdot \sqrt{500^3}} - \frac{(28 - 0)}{(28 - 0) + 0.04 \cdot \sqrt{500^3}}$$

$$\beta_{ds} = 0.99955 - 0.05892 = 0.94063$$

3.1.4 (6): Tab. 3.3: k_h coefficient depending on h_0

$k_h = 0.70$ for $h_0 \geq 500\ mm$

Annex B.2 (1): Eq. B.11: $\epsilon_{cd,0}$ basic drying shrinkage strain

$$\epsilon_{cd,0} = 0.85 \left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}} \right) \right] \cdot 10^{-6} \cdot \beta_{RH}$$

Annex B.2 (1): Eq. B.12: β_{RH}
RH the ambient relative humidity (%)

$$\beta_{RH} = 1.55 \left[1 - \left(\frac{RH}{RH_0} \right)^3 \right] = 1.55 \left[1 - \left(\frac{80}{100} \right)^3 \right] = 0.7564$$

$$\epsilon_{cd,0} = 0.85\left[(220 + 110 \cdot 4) \cdot \exp\left(-0.12 \cdot \frac{43}{10}\right)\right] \cdot 10^{-6} \cdot 0.7564$$

Annex B.2 (1): $\alpha_{ds1}, \alpha_{ds1}$ coefficients depending on type of cement.
For class N $\alpha_{ds1} = 4$, $\alpha_{ds2} = 0.12$

$$\epsilon_{cd,0} = 2.533 \cdot 10^{-4}$$

$$\epsilon_{cd} = 0.94063 \cdot 0.70 \cdot 2.533 \cdot 10^{-4} = 0.0001668$$

$$\epsilon_{cd} = 1.668 \cdot 10^{-4} = 0.1668\ ‰$$

$$\epsilon_{ca}(t) = \beta_{as}(t) \cdot \epsilon_{ca}(\infty)$$

3.1.4 (6): Eq. 3.11: ϵ_{ca} autogenous shrinkage strain

$$\epsilon_{ca}(\infty) = 2.5\,(f_{ck} - 10) \cdot 10^{-6} = 2.5\,(35 - 10) \cdot 10^{-6}$$

3.1.4 (6): Eq. 3.12: $\epsilon_{ca}(\infty)$

$$\epsilon_{ca}(\infty) = 6.25 \cdot 10^{-5} = 0.0625\ ‰$$

Proportionally to $\beta_{ds}(t, ts)$, SOFiSTiK calculates factor β_{as} as follows:

$$\beta_{as} = \beta_{as}(t) - \beta_{as}(t_0)$$

3.1.4 (6): Eq. 3.13: β_{as}

$$\beta_{as} = 1 - e^{-0.2 \cdot \sqrt{t}} - \left(1 - e^{-0.2 \cdot \sqrt{t_0}}\right) = e^{-0.2 \cdot \sqrt{t_0}} - e^{-0.2 \cdot \sqrt{t}}$$

$$\beta_{as} = 0.347$$

$$\epsilon = \epsilon_{cd,0} + \epsilon_{ca}(\infty) = 2.533 \cdot 10^{-4} + 6.25 \cdot 10^{-5}$$

$$\epsilon = -31.58 \cdot 10^{-5}$$

ϵ absolute shrinkage strain
negative sign to declare losses

$$\epsilon_{ca} = 0.347 \cdot 6.25 \cdot 10^{-5} = 2.169 \cdot 10^{-5} = 0.02169\ ‰$$

$$\epsilon_{cs} = 1.668 \cdot 10^{-4} + 2.169 \cdot 10^{-5} = -18.85 \cdot 10^{-5}$$

negative sign to declare losses

$$\phi(t, t_0) = \phi_0 \cdot \beta_c(t, t_0)$$

Annex B.1 (1): Eq. B.1: $\phi(t, t_0)$ creep coefficient

$$\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

Annex B.1 (1): Eq. B.2: ϕ_0 notional creep coefficient

$$\phi_{RH} = \left[1 + \frac{1 - RH/100}{0.1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1\right] \cdot \alpha_2$$

Annex B.1 (1): Eq. B.3: ϕ_{RH} factor for effect of relative humidity on creep

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = 16.8/\sqrt{43} = 2.562$$

Annex B.1 (1): Eq. B.4: $\beta(f_{cm})$ factor for effect of concrete strength on creep

$$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7} \leq 1 = 0.8658$$

Annex B.1 (1): Eq. B.8c: α_1, α_2, α_3 coefficients to consider influence of concrete strength

$$\alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2} \leq 1 = 0.9597$$

$$\alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5} \leq 1 = 0.9022$$

$$\phi_{RH} = \left[1 + \frac{1 - 80/100}{0.1 \cdot \sqrt[3]{500}} \cdot 0.8658\right] \cdot 0.9597 = 1.1691$$

$$\beta(t_0) = \frac{1}{\left(0.1 + t_0^{0.20}\right)}$$

Annex B.1 (1): Eq. B.5: $\beta(t_0)$ factor for effect of concrete age at loading on creep

Annex B.1 (2): Eq. B.9: $t_{0,T}$ temperature adjusted age of concrete at loading adjusted according to expression B.10

Annex B.1 (3): Eq. B.10: t_T temperature adjusted concrete age which replaces t in the corresponding equations

$$t_0 = t_{0,T} \cdot \left(\frac{9}{2 + t_{0,T}^{1.2}} + 1 \right)^{\alpha} \geq 0.5$$

$$t_T = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)]-13.65)} \cdot \Delta t_i$$

$$t_{0,T} = 28 \cdot e^{-(4000/[273+20]-13.65)} = 28 \cdot 1.0 = 28.0$$

Annex B.1 (2): Eq. B.9: α a power which depends on type of cement
For class N $\alpha = 0$

$$\Rightarrow t_0 = 28.0 \cdot \left(\frac{9}{2 + 28.0^{1.2}} + 1 \right)^{0} = 28.0$$

$$\beta(t_0) = \frac{1}{(0.1 + 28.0^{0.20})} = 0.48844$$

Annex B.1 (1): Eq. B.7: $\beta_c(t, t_0)$ coefficient to describe the development of creep with time after loading

Annex B.1 (1): Eq. B.8: β_H coefficient depending on relative humidity and notional member size

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{(\beta_H + t - t_0)} \right]^{0.3}$$

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot RH)^{18} \right] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3$$

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot 80)^{18} \right] \cdot 500 + 250 \cdot 0.9022$$

$$\beta_H = 1335.25 \leq 1500 \cdot 0.9022 = 1353.30$$

$$\Rightarrow \beta_c(t, t_0) = 0.9996$$

$$\phi_0 = 1.1691 \cdot 2.562 \cdot 0.48844 = 1.463$$

Annex B.1 (3): The values of $\phi(t, t_0)$ given above should be associated with the tangent modulus E_c

3.1.4 (2): The values of the creep coefficient, $\phi(t, t_0)$ is related to E_c, the tangent modulus, which may be taken as $1.05 \cdot E_{cm}$

$$\phi(t, t_0) = 1.463 \cdot 0.9996 / 1.05 = 1.393$$

According to EN, the creep value is related to the tangent Young's modulus E_c, where E_c being defined as $1.05 \cdot E_{cm}$. To account for this, SOFiSTiK adopts this scaling for the computed creep coefficient (in SOFiSTiK, all computations are consistently based on E_{cm}).

5.10.6 (2): Eq. 5.46: ΔP_{c+s+r} time dependent losses of prestress

$$\Delta P_{c+s+r} = A_p \cdot \Delta\sigma_{p,c+s+r} = A_p \frac{\epsilon_{cs} \cdot E_p + 0.8 \Delta\sigma_{pr} + \dfrac{E_p}{E_{cm}} \phi(t, t_0) \cdot \sigma_{c,QP}}{1 + \dfrac{E_p}{E_{cm}} \dfrac{A_p}{A_c} \left(1 + \dfrac{A_c}{I_c} z_{cp}^2 \right) [1 + 0.8 \phi(t, t_0)]}$$

In this example only the losses due to creep and shrinkage are taken into account, the reduction of stress due to relaxation ($\Delta\sigma_{pr}$) is ignored.

$$\Delta\sigma_{p,c+s} = \frac{-0.1885 \cdot 10^{-3} \cdot 195000 + 5.7223 \cdot 1.393 \cdot (-4.82)}{1 + 5.7223 \dfrac{28.5 \cdot 10^{-4}}{0.9926} \left(1 + \dfrac{0.9926}{0.08214} 0.3901^2 \right) [1 + 0.8 \cdot 1.393]}$$

$\Delta\sigma_{p,c+s}$ variation of stress in tendons due to creep and shrinkage at location x, at time t

$$\Delta\sigma_{p,c+s} = -68.46 \ MPa$$

$$\Delta P_{c+s} = A_p \cdot \Delta\sigma_{p,c+s} = 28.5 \cdot 10^{-4} \cdot 68.46 \cdot 10^3 = 195.11 \ kN$$

5 Conclusion

This example shows the calculation of the time dependent losses due to creep and shrinkage. It has been shown that the results are in very good agreement with the reference solution.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.