Benchmark Example No. 13

Design of a Steel I-section for Bending and Shear

SOFiSTiK | 2023

VERiFiCATiON
DCE-EN13 Design of a Steel I-section for Bending and Shear

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Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview

Design Code Family(s):	EN
Design Code(s):	EN 1993-1-1
Module(s):	AQB, AQUA
Input file(s):	usage_steel.dat

1 Problem Description

The problem consists of a steel I- section, as shown in Fig. 1. The cross-section is designed for bending and shear.

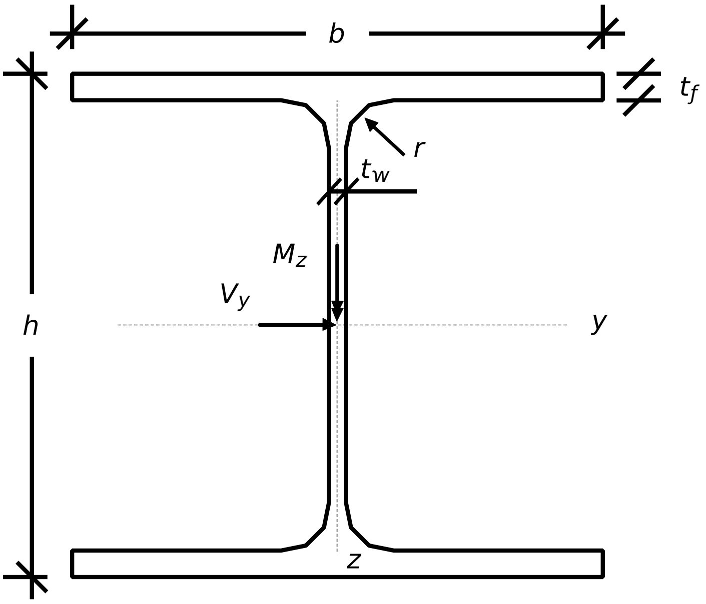

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the resistance of steel cross-sections for bending and shear. The content of this problem is covered by the following parts of EN 1993-1-1:2005 [1]:

- Structural steel (Section 3.2)
- Resistance of cross-sections (Section 6.2)

3 Model and Results

The I-section, a HEA 200, with properties as defined in Table 1, is to be designed for an ultimate moment M_z and a shear force V_y, with respect to EN 1993-1-1:2005 [1]. The calculation steps are presented below and the results are given in Table 2. The utilisation level of allowable plastic forces are calculated and compared to SOFiSTiK results.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$S\ 235$	HEA 200 $b = 20.0\ cm$ $h = 19.0\ cm$ $t_f = 1.0\ cm$, $t_w = 0.65\ cm$ $r = 1.8\ cm$	$V_y = 200$ or $300\ kN$ $M_z = 20\ kNm$

Table 2: Results

Case	Result	SOF. (EC3 Tables)	SOF. (FEM)	Ref.
	$V_{pl,Rd,y}$ $[kN]$	542.71	611.42	542.71
	$V_{pl,Rd,z}$ $[kN]$	245.32	228.67	245.32
I	Util. level V_y	0.369	0.327	0.369
	Util. level M_z	0.418	0.418	0.418
II	Util. level V_y	0.553	0.491	0.553
	Util. level M_z	0.418	0.418	0.418

4 Design Process[1]

Material:

Structural Steel S 235

$f_y = 235\ N/mm^2$

Tab. 3.1 : Nominal values of yield strength f_y and ultimate tensile strength f_u for hot rolled structural steel.

Cross-sectional properties:

$W_{pl,y} = 429.5\ cm^3$

$W_{pl,z} = 203.8\ cm^3$

Plastic section modulus of HEA 200 w.r.t. $y-$ and $z-$axis

Where the shear force is present allowance should be made for its effect on the moment resistance. This effect may be neglected, where the shear force is less than half the plastic shear resistance.

6.2.8: Bending and shear

$V_{Ed} \leq 0.5\ V_{pl,Rd}$

6.2.8 (2)

$$V_{pl,Rd} = \frac{A_V \cdot \left(f_y \,/\, \sqrt{3}\right)}{\gamma_{M0}}$$

6.2.6 (2): Eq. 6.18: $V_{pl,Rd}$ the design plastic shear resistance

A_V the shear area w.r.t. $y-$ and $z-$axis, respectively

$A_{V_y} = 2 \cdot A_{flange}$ (in the $y-$direction only contribution the two flanges)

$A_{V_y} = 2 \cdot t_f \cdot b = 2 \cdot 1 \cdot 20 = 40\ cm^2$

$A_{V_z} = A - 2 \cdot b \cdot t_f + (t_w + 2 \cdot r) \cdot t_f$ but not less than $\eta \cdot h_w \cdot t_w$

$A_{V_z} = 53.8 - 2 \cdot 20 \cdot 1 + (0.65 + 2 \cdot 1.8) \cdot 1$

6.2.6 (3): The shear area A_V may be taken as follows for rolled I-sections with load parallel to the web

$h_w = h - 2 \cdot t_f = 17\ cm$

$A_{V_z} = 18.08\ cm^2 > 1 \cdot 17 \cdot 0.65 = 11.05$

6.2.6 (3): η may be conservatively taken equal to 1.0

6.1 (1): Partial factor $\gamma_{M0} = 1.00$ is recommended

$$V_{pl,Rd,y} = \frac{40 \cdot \left(23.5 \,/\, \sqrt{3}\right)}{1.00} = 542.70\ kN$$

$$V_{pl,Rd,z} = \frac{18.08 \cdot \left(23.5 \,/\, \sqrt{3}\right)}{1.00} = 245.30\ kN$$

a) Finite Element Method:

- **Case** I:

According to FEM analysis:

$V_{pl,Rd,y} = 611.85\ kN$

$V_{pl,Rd,z} = 228.38\ kN$

we have:

$V_{Ed} = V_y = 200\ kN,\ M_z = 20\ kNm$

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[1] The sections mentioned in the margins refer to EN 1993-1-1:2005 [1] unless otherwise specified.

$$\frac{V_{Ed}}{V_{pl,Rd,y}} = \frac{200}{611.85} = 0.3268 < 0.5$$

→ no reduction of moment resistance due to shear

6.2.8 (5): Eq. 6.30: The reduced design plastic resistance moment (for $\rho = 0$)

$$M_{z,V,Rd} = \frac{W_{pl,z} \cdot f_y}{\gamma_{M0}} = \frac{203.8 \cdot 23.5}{1} = 4789\ kNcm\ =\ 47.89\ kNm$$

$$\frac{M_z}{M_{z,V,Rd}} = \frac{20}{47.89} = 0.418$$

- **Case** II:

$V_{Ed} = V_y = 300\ kN$, $M_z = 20\ kNm$

$$\frac{V_{Ed}}{V_{pl,Rd,y}} = \frac{300}{611.85} = 0.490 < 0.5$$

→ no reduction of moment resistance due to shear

$$M_{z,V,Rd} = \frac{W_{pl,z} \cdot f_y}{\gamma_{M0}}$$

$$M_{z,V,Rd} = 47.89 kNm$$

$$\frac{M_z}{M_{z,V,Rd}} = \frac{20}{47.89} = 0.418$$

b) EC3 Tables:

$V_{pl,Rd,y} = 542.71\ kN$

$V_{pl,Rd,z} = 245.32\ kN$

- **Case** I:

$V_{Ed} = V_y = 200\ kN$, $M_z = 20\ kNm$

$$\frac{V_{Ed}}{V_{pl,Rd,y}} = \frac{200}{542.71} = 0.368 < 0.5$$

→ no reduction of moment resistance due to shear

6.2.8 (5): Eq. 6.30: The reduced design plastic resistance moment (for $\rho = 0$)

$$M_{z,V,Rd} = \frac{W_{pl,z} \cdot f_y}{\gamma_{M0}} = \frac{203.8 \cdot 23.5}{1} = 4789\ kNcm\ =\ 47.89\ kNm$$

$$\frac{M_z}{M_{z,V,Rd}} = \frac{20}{47.89} = 0.418$$

- **Case** II:

$V_{Ed} = V_y = 300\ kN$, $M_z = 20\ kNm$

$$\frac{V_{Ed}}{V_{pl,Rd,y}} = \frac{300}{542.71} = 0.552 > 0.5$$

6.2.8 (3): The reduced moment resistance should be taken as the design resistance of the cross-section, calculated using a reduced yield strength $(1 - \rho) \cdot f_y$ for the shear area

→ reduction of moment resistance due to shear

$$\begin{aligned}\rho &= \left(\frac{2\cdot V_{Ed}}{V_{pl,Rd,y}}-1\right)^2\\ &= \left(\frac{2\cdot 300}{542.71}-1\right)^2\\ &= 0.011143\end{aligned}$$

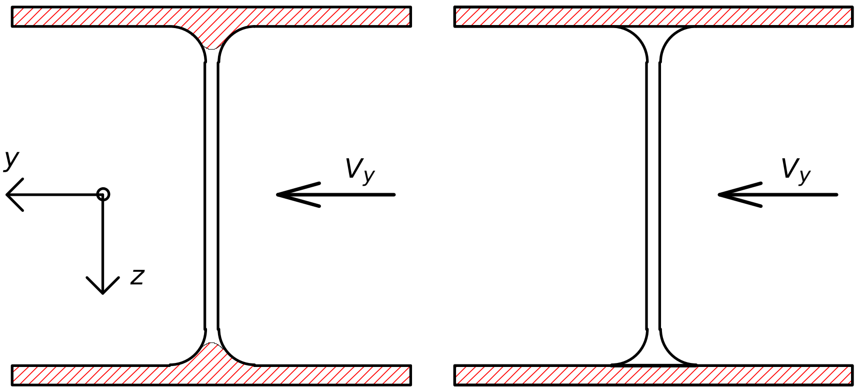
$$M_{z,V,Rd} = (1-\rho)\cdot\frac{W_{pl,z}\cdot f_y}{\gamma_{M0}}$$

$$M_{z,V,Rd} = (1-0.0111435)\cdot 47.89 = 47.35\ kNm$$

$$\frac{M_z}{M_{z,V,Rd}} = \frac{20}{47.356} = 0.4223$$

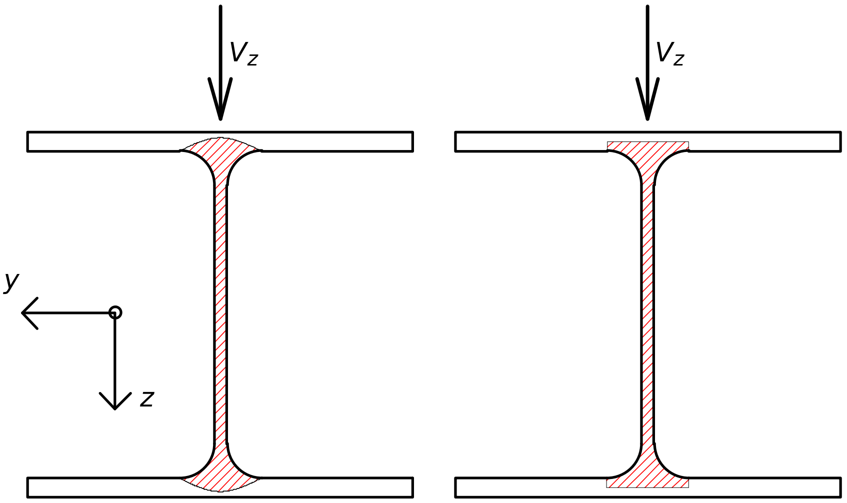
5 Conclusion

This example shows the calculation of the resistance of steel cross-section for bending and shear. It has been shown that the results are reproduced with excellent accuracy.



a) FEM Analysis (SOFiSTiK)   a) Reference

Figure 2: Visualisation of areas and approach for calculating the $V_{pl,Rd,y}$



a) FEM Analysis (SOFiSTiK)   a) Reference

Figure 3: Visualisation of areas and approach for calculating the $V_{pl,Rd,z}$

By using FEM analysis the $V_{pl,Rd,y}$ and $V_{pl,Rd,z}$ values are deviating from the reference values. The reason behind this difference is shown in Fig. 2, 3, 4 and 5.

FEM analysis represents a real physical behaviour, because as shown in Fig. 2 and 3 the calculated area is taken approximately for the reference $V_{pl,Rd,y}$ and $V_{pl,Rd,z}$ values. The results by using FEM analysis are more accurate in reality than the reference example.

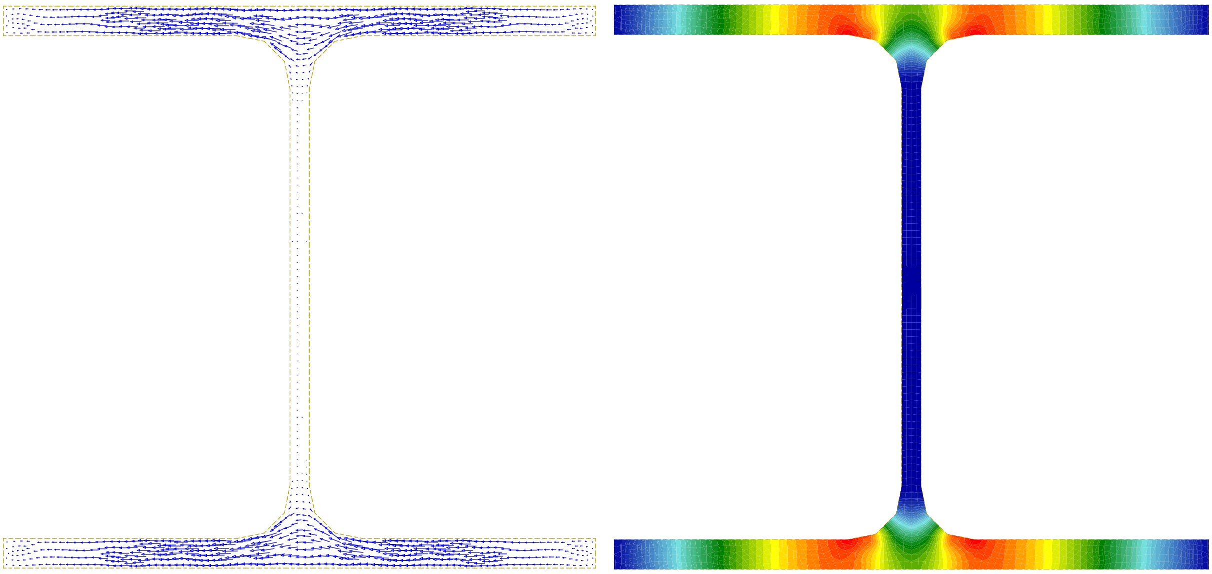


Figure 4: $V_{pl,Rd,y}$ - SOFiSTiK Results (FEM)

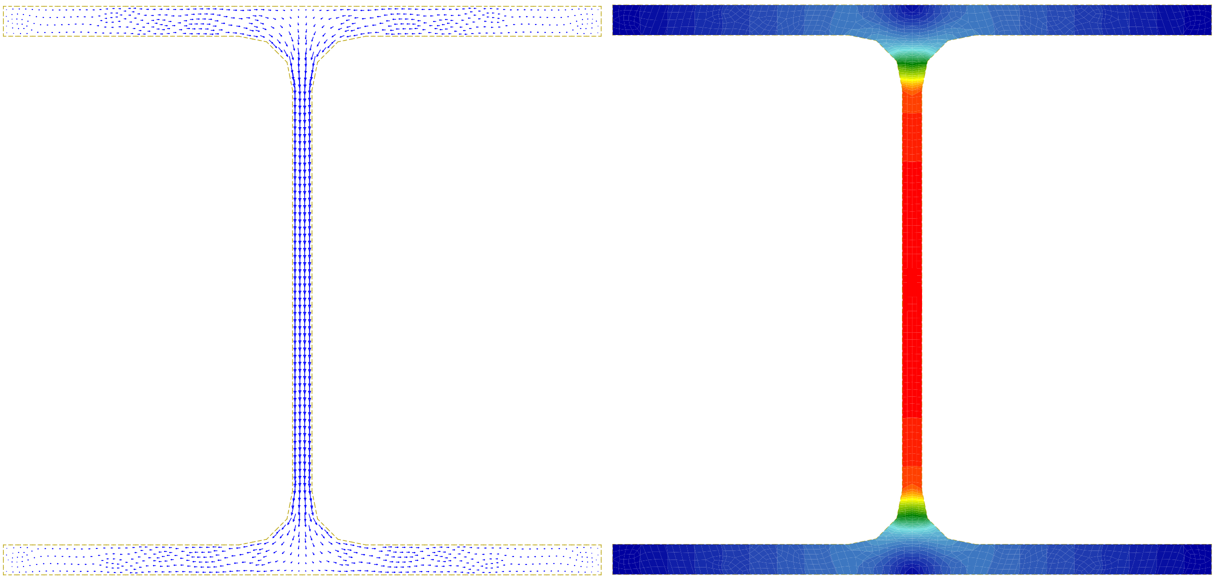


Figure 5: $V_{pl,Rd,z}$ - SOFiSTiK Results (FEM)

6 Literature

[1] *EN 1993-1-1: Eurocode 3: Design of steel structures, Part 1-1: General rules and rules for buildings*. CEN. 2005.

[2] Schneider. *Bautabellen für Ingenieure*. 21th. Bundesanzeiger Verlag, 2014.