Benchmark Example No. 12

Crack width calculation of reinforced beam acc. DIN EN 1992-1-1

SOFiSTiK | 2023

VERiFiCATiON
DCE-EN12 Crack width calculation of reinforced beam acc. DIN EN 1992-1-1

VERiFiCATiON Manual, Service Pack 2023-10 Build 44

SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	crack_widths.dat

1 Problem Description

The problem consists of a rectangular section, asymmetrically reinforced, as shown in Fig. 1. Different loading conditions are examined, always consisting of a bending moment M_{Ed}, and in addition with or without a compressive or tensile axial force N_{Ed}. The crack width is determined.

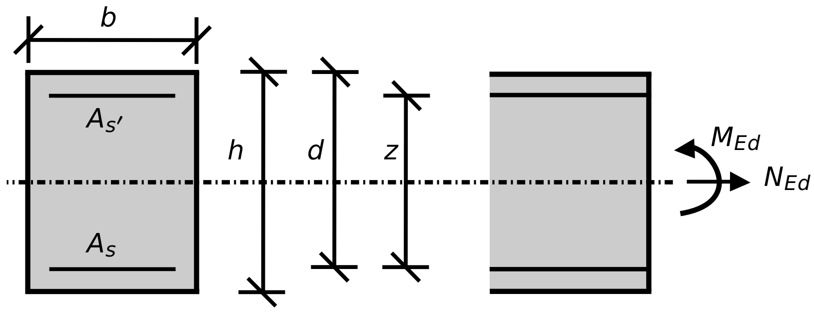


Figure 1: Problem Description

2 Reference Solution

This example is concerned with the control of crack widths. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Basic assumptions for calculation of crack widths (Section 7.3.2, 7.3.3, 7.3.4)

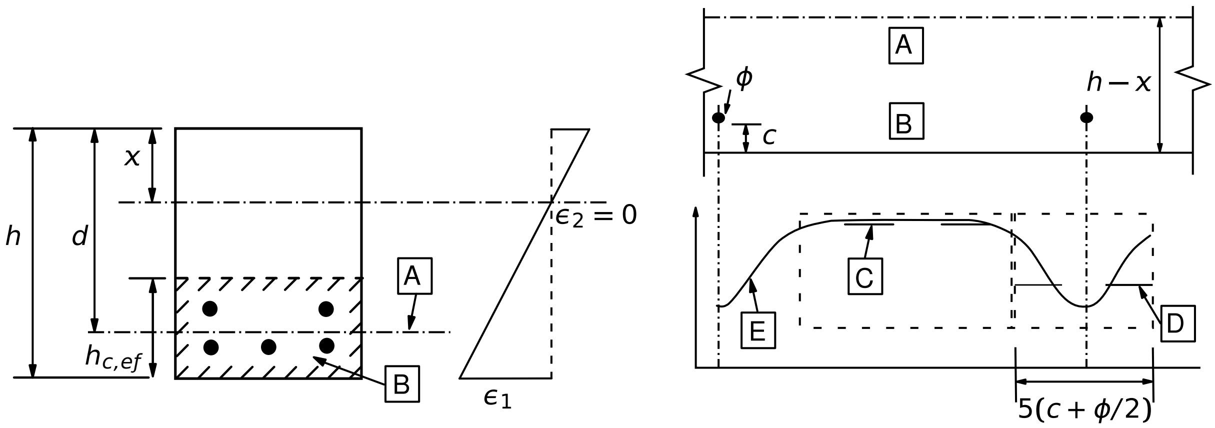


Figure 2: Stress and Strain Distributions in the Design of Cross-sections

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed for crack width, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1] [2]. The calculation steps with different loading conditions and calculated with different sections of DIN EN 1992-1-1:2004 are presented below and the results are given in Table 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\,25/30$	$h = 100.0\,cm$	$N_{Ed} = 0$ or $\pm 300\,kN$
$B\,500A$	$d = 96.0\,cm$	$M_{Ed} = 562.5\,kNm$
	$b = 30.0\,cm$	
	$\phi_s = 25.0\,mm, A_s = 24.50\,cm^2$	
	$\phi_{s'} = 12.0\,mm, A_{s'} = 2.26\,cm^2$	
	$w_k = 0.3\,mm$	

Table 2: Results

Case	Load	A_s given $[cm^2]$	Result	SOF.	Ref.
I	$M_{Ed}, N_{Ed} = 0$	24.50	$A_{s,requ}$ $[cm^2]$	6.93	6.93
			σ_s $[MPa]$	207.85	207.85
II	$M_{Ed}, N_{Ed} = 300$	24.50	$A_{s,requ}$ $[cm^2]$	10.39	10.39
III	$M_{Ed}, N_{Ed} = -300$	24.50	$A_{s,requ}$ $[cm^2]$	4.04	4.04
IV	$M_{Ed}, N_{Ed} = 0$	24.50	A_s $[cm^2]$		passed with given reinforcement
			σ_s $[MPa]$	440.57	440.53
V	$M_{Ed}, N_{Ed} = 0$	12.0	A_s $[cm^2]$		not passed with given reinforcement
		→ new: 14.54	σ_s $[MPa]$	436.30	436.28
VI	$M_{Ed}, N_{Ed} = 0$	24.50	w_k $[mm]$	0.13	0.13

4 Design Process [1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

Concrete: $\gamma_c = 1.50$
Steel: $\gamma_s = 1.15$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

$f_{ck} = 25\ MPa$

Tab. 3.1 : Strength for concrete

$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 25/1.5 = 14.17\ MPa$

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

3.2.7: (2), Fig. 3.8

Design Load:

$M_{Ed} = 562.5\ kNm$

$N_{Ed} = 0.0$ or $\pm 300\ kN$

Minimum reinforcement areas

7.3.2: If crack control is required, a minimum amount of bonded reinforcement is required to control cracking in areas where tension is expected

- *Case I*: $M_{Ed}, N_{Ed} = 0.0$

$A_{s,min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}$

7.3.2 (2): Eq. 7.1: $A_{s,min}$ minimum area of reinforcing steel within tensile zone

$f_{ct,eff} = f_{ctm} \geq 3.0\ MPa$

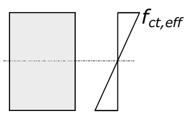
(NDP) 7.3.2 (2): $f_{ct,eff}$ mean value of concrete tensile strength
When the cracking time can't be placed with certainty in the first 28 days then $f_{ct,eff} \geq 3.0\ MPa$ for normal concrete

$f_{ct,eff} = 2.6 < 3.0\ MPa \ \rightarrow f_{ct,eff} = 3.0\ MPa$

Tab. 3.1: $f_{ctm} = 2.6\ MPa$ for C 25

$A_{ct} = b \cdot h_{eff} = 0.3 \cdot 0.5 = 0.15\ m^2$

7.3.2 (2): A_{ct} is the area of concrete within tensile zone, for pure bending of a rectangular section its height is $h/2$



$h_{eff} = h/2$

7.3.2 (2): Eq. 7.2: k_c for bending of rectangular sections

$$k_c = 0.4 \cdot \left[1 - \frac{\sigma_c}{k_1 \cdot (h/h^*) \cdot f_{ct,eff}}\right] \leq 1$$

7.3.2 (2): Eq. 7.4: σ_c mean stress of the concrete

$\sigma_c = N_{Ed}/(b \cdot h) = 0.0 \rightarrow \ k_c = 0.4$

7.3.2 (2): h is the lesser value of b, h
$h = \min\{300, 1000\} = 300\ mm$

$k = 0.8$ for $h \leq 300$

$\phi_s = \phi_s^* \cdot f_{ct,eff}\ /\ 2.9\ N/mm^2$

$\phi_s^* = 25 \cdot 2.9\ /\ 3.0 = 24.16667$

7.3.2 (NA.5): Eq. NA.7.5.2: For determination of the reinforcement stress the maximum bar diameter has to be modified

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8.
[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

(NDP) 7.3.3 Tab. 7.2DE (a) where $w_k = 0.3\ m$ the prescribed maximum crack width

$$\sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 \ / \ \phi_s^*} = 207.846\ MPa$$

$$A_{s,requ} = 0.4 \cdot 0.8 \cdot 3.0 \cdot 0.15 \cdot 10^4 / 207.846 = 6.928 cm^2$$

- *Case II*: $N_{Ed} = 300\ kN$

7.3.2 (2): Eq. 7.1: $A_{s,min}$ minimum area of reinforcing steel within tensile zone

$$A_{s,min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}$$

$$f_{ct,eff} = f_{ctm} \geq 3.0\ MPa$$

Tab. 3.1: $f_{ctm} = 2.6\ MPa$ for C 25

$$f_{ct,eff} = 2.6 < 3.0\ MPa \ \rightarrow f_{ct,eff} = 3.0\ MPa$$

7.3.2 (2): A_{ct} is the area of concrete within tensile zone

$$A_{ct} = b \cdot h_{eff} = 0.15\ m^2$$

7.3.2 (2): Eq. 7.4: σ_c mean stress of the concrete, tensile stress $\sigma_c < 0$

$$\sigma_c = \frac{N_{Ed}}{b \cdot h} = \frac{300 \cdot 10^3}{300 \cdot 1000} = 1.0\ MPa$$

7.3.2 (2): h is the lesser value of b, h
$h = \min\{300, 1000\} = 300\ mm$

$$k = 0.8 \text{ for } h \leq 300$$

(NDP) 7.3.2 (NA.5): Eq. NA.7.5.2: For determination of the reinforcement stress the maximum bar diameter has to be modified

$$\phi_s = \phi_s^* \cdot f_{ct,eff} \ / \ 2.9\ N/mm^2$$

$$\phi_s^* = 25 \cdot 2.9 \ / \ 3.0 = 24.16667$$

(NDP) 7.3.3 Tab. 7.2DE (a) where $w_k = 0.3\ m$ the prescribed maximum crack width

$$\sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 \ / \ \phi_s^*} = 207.846\ MPa$$

7.3.2 (2): Eq. 7.2: k_c for bending with axial force of rectangular sections

$$k_c = 0.4 \cdot \left[1 - \frac{\sigma_c}{k_1 \cdot (h/h^*) \cdot f_{ct,eff}}\right] \leq 1$$

$h^* = 1.0\ m$ for $h \geq 1.0\ m$
$k_1 = 2 \cdot h^* \ / \ (3 \cdot h)$ if N_{Ed} tensile force

$$k_1 = 2 \cdot h^* \ / \ (3 \cdot h) = 2/3$$

7.3.2 (2): Eq. 7.1: Tensile stress $\sigma_c < 0$

$$k_c = 0.4 \cdot \left[1 + \frac{1.0}{(2/3) \cdot 1 \cdot 3.0}\right] = 0.6 \leq 1$$

$$A_{s,requ} = 0.6 \cdot 0.8 \cdot 3.0 \cdot 0.15 \cdot 10^4 / 207.846 = 10.39 cm^2$$

- *Case III*: $N_{Ed} = -300\ kN$

7.3.2 (2): Eq. 7.1: $A_{s,min}$ minimum area of reinforcing steel within tensile zone

$$A_{s,min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}$$

$$f_{ct,eff} = f_{ctm} \geq 3.0\ MPa$$

Tab. 3.1: $f_{ctm} = 2.6\ MPa$ for C 25

$$f_{ct,eff} = 2.6 < 3.0\ MPa \ \rightarrow f_{ct,eff} = 3.0\ MPa$$

7.3.2 (2): A_{ct} is the area of concrete within tensile zone

$$A_{ct} = b \cdot h_{eff}$$

The height of the tensile zone is determined through the stresses:

7.3.2 (2): Eq. 7.4: σ_c mean stress of the concrete, compressive stress $\sigma_c > 0$

$$\sigma_c = \frac{N_{Ed}}{b \cdot h} = \frac{300 \cdot 10^3}{300 \cdot 1000} = 1.0\ MPa$$

$$\sigma_u = f_{ct,eff} = 3.0\ MPa$$

$$\Rightarrow \ h_{eff} = \frac{3.0 \cdot 50}{3.0 + 1.0} = 37.5 cm$$

$$A_{ct} = 0.3 \cdot 0.375 = 0.1125\ m^2$$

7.3.2 (2): h is the lesser value of b, h
$h = \min\{300, 1000\} = 300\ mm$

$$k = 0.8 \text{ for } h \leq 300$$

(NDP) 7.3.2 (NA.5): Eq. NA.7.5.2: For determination of the reinforcement stress the maximum bar diameter has to be modified

$\phi_s = \phi_s^* \cdot f_{ct,eff} \; / \; 2.9 \; N/mm^2$

$\phi_s^* = 25 \cdot 2.9 \; / \; 3.0 = 24.16667$

$\sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 \; / \; \phi_s^*} = 207.846 \; MPa$

(NDP) 7.3.3 Tab. 7.2DE (a) where $w_k = 0.3 \; m$ the prescribed maximum crack width

$k_c = 0.4 \cdot \left[1 - \frac{\sigma_c}{k_1 \cdot (h/h^*) \cdot f_{ct,eff}} \right] \leq 1$

7.3.2 (2): Eq. 7.2: k_c for bending with axial force of rectangular sections

$k_1 = 1.5$

$h^* = 1.0 \; m$ for $h \geq 1.0 \; m$
$k_1 = 1.5$ if N_{Ed} compressive force

$k_c = 0.4 \cdot \left[1 - \frac{1.0}{1.5 \cdot 1 \cdot 3.0} \right] = 0.3111 \leq 1$

$A_{s,requ} = 0.3111 \cdot 0.8 \cdot 3.0 \cdot 0.1125 \cdot 10^4 / 207.846 = 4.04 \; cm^2$

Control of cracking without direct calculation

7.3.3: Control of cracking without direct calculation
Examples calculated in this section are w.r.t. Table 7.2DE, here Table 7.3N is not relevant

- *Case IV*: $N_{Ed} = 0.0$

(NDP) 7.3.2 (2): $f_{ct,eff}$ mean value of concrete tensile strength

$f_{ct,eff} = f_{ctm} \geq 3.0 \; MPa$

Tab. 3.1: $f_{ctm} = 2.6 \; MPa$ for C 25

$f_{ct,eff} = 2.6 < 3.0 \; MPa \; \rightarrow f_{ct,eff} = 3.0 \; MPa$

(NDP) 7.3.3: Eq. 7.7.1DE: The maximum bar diameters should be modified for load action

$\phi_s = \phi_s^* \cdot \frac{\sigma_s \cdot A_s}{4(h-d) \cdot b \cdot 2.9} \geq \phi_s^* \cdot \frac{f_{ct,eff}}{2.9}$

$A_s = 24.5 \; cm^2$ prescribed reinforcement

$\phi_s = 25 \; mm = \phi_s^* \cdot \frac{264.06 \cdot 24.50}{4(100-96) \cdot 30 \cdot 2.9} = \phi_s^* \cdot 4.6476$

$\rightarrow \; \phi_s^* = 5.3791 \; mm$

$\sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 \; / \; \phi_s^*} = 440.53 \; MPa$

(NDP) 7.3.3 Tab. 7.2DE (a) where $w_k = 0.3 \; m$ the prescribed maximum crack width

$\sigma_s = 264.06 < 440.53 \; MPa$

which corresponds to the value calculated in SOFiSTiK [ssr]

→ crack width control is passed with given reinforcement.

In case the usage factor becomes 1.0 then the stresses σ_s are equal, as it can be seen in Case *V* below.

and $\phi_s = 25 \; mm > \phi_s^* \cdot \frac{f_{ct,eff}}{2.9} = 5.3791 \cdot \frac{3.0}{2.9} = 5.5646$

also control the steel stress with respect to the calculated strains

$\epsilon_s = 0.440 + 1.913 \cdot (0.50 - 0.04) = 1.31998$

$\rightarrow \; \sigma_s = 0.00131998 \cdot 200000 = 264.0 \; MPa$

or

from Tab. 7.2DE and for $\sigma_s = 264.04 \approx 260.0 \; MPa$

(NDP) 7.3.3 Tab. 7.2DE (a)

$\rightarrow \; \phi_s^* = 15 \; mm \; \rightarrow \phi_s = \phi_s^* \cdot \frac{\sigma_s \cdot A_s}{4(h-d) \cdot b \cdot 2.9} \geq \phi_s^* \cdot \frac{f_{ct,eff}}{2.9}$

(NDP) 7.3.3: Eq. 7.7.1DE: The maximum bar diameters should be modified for load action

$$\phi_s = 15 \cdot \frac{264.06 \cdot 24.50}{4(100-96) \cdot 30 \cdot 2.9} = 69.69\ mm > 25\ mm$$

→ crack width control is passed with given reinforcement.

- $Case\ \ V$: $N_{Ed} = 0.0$, $A_s = 12.0\ cm^2$

In this case, the prescribed reinforcement is decreased from $A_s = 24.5\ cm^2$ to $A_s = 12.0\ cm^2$ in order to examine a case where the the crack width control is not passed with the given reinforcement.

(NDP) 7.3.2 (2): $f_{ct,eff}$ mean value of concrete tensile strength

$f_{ct,eff} = f_{ctm} \geq 3.0\ MPa$

Tab. 3.1: $f_{ctm} = 2.6\ MPa$ for C 25

$f_{ct,eff} = 2.6 < 3.0\ MPa\ \rightarrow f_{ct,eff} = 3.0\ MPa$

(NDP) 7.3.3 Tab. 7.2DE (a)

from Tab. 7.2DE and for $\sigma_s = 509.15 \approx 510.0\ MPa$

(NDP) 7.3.3: Eq. 7.7.1DE: The maximum bar diameters should be modified for load action

$$\rightarrow\ \phi_s^* \approx 3.9\ mm\ \rightarrow \phi_s = \phi_s^* \cdot \frac{\sigma_s \cdot A_s}{4(h-d) \cdot b \cdot 2.9}$$

$$\phi_s = 3.9 \cdot \frac{509.15 \cdot 12.0}{4(100-96) \cdot 30 \cdot 2.9} = 17.12\ mm < 25\ mm$$

→ crack width control is not passed with given reinforcement.

→ start increasing reinforcement in order to be in the limits of admissible steel stresses

(NDP) 7.3.3 Tab. 7.2DE (a)

from Tab. 7.2DE and for $\sigma_s = 436.43\ MPa$

7.3.3: Eq. 7.7.1DE: The maximum bar diameters should be modified for load action

$$\rightarrow\ \phi_s^* \approx 5.6\ mm\ \rightarrow \phi_s = \phi_s^* \cdot \frac{\sigma_s \cdot A_s}{4(h-d) \cdot b \cdot 2.9}$$

$$\phi_s = 5.6 \cdot \frac{436.43 \cdot 14.54}{4(100-96) \cdot 30 \cdot 2.9} = 25.45\ mm \geq 25\ mm$$

→ crack width control passed with additional reinforcement.

If we now input as prescribed reinforcement the reinforcement that is calculated in order to pass crack control, i.e. $A_s = 14.54\ cm^2$ we get a steel stress of $\sigma_s = 436.36\ MPa$ which gives

$$\phi_s = 25\ mm = \phi_s^* \cdot \frac{436.36 \cdot 14.54}{4(100-96) \cdot 30 \cdot 2.9}$$

$$\rightarrow\ \phi_s^* = 5.4849\ mm$$

(NDP) 7.3.3 Tab. 7.2DE (a) where $w_k = 0.3\ m$ the prescribed maximum crack width

$$\sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6\ /\ \phi_s^*}$$

$$\sigma_s = \sqrt{0.3 \cdot 3.48 \cdot 10^6\ /\ 5.4849} = 436.28\ MPa$$

Here we can notice that the stresses are equal leading to a usage factor of 1.0

7.3.4: Control of cracking with direct calculation

Control of cracking with direct calculation

- *Case VI*: $N_{Ed} = 0.0$

$\alpha_e = E_s \,/\, E_{cm} = 200000 \,/\, 31476 = 6.354$

7.3.4 (1): α_e is the ratio $E_s \,/\, E_{cm}$

$\rho_{p,eff} = (A_s + \xi_1^2 \cdot A'_p) \,/\, A_{c,eff}$

7.3.4 (1): Eq. 7.10: where A'_p and $A_{c,eff}$ are defined in 7.3.2 (3)

$A_{c,eff} = h_{c,ef} \cdot b$

7.3.2 (3): $h_{c,ef}$ see Fig. 7.1DE (d)

$h \,/\, d_1 = 100/4 = 25.00 \;\rightarrow\; h_{c,ef} \,/\, d_1 = 3.25$

$A_{c,eff} = (3.25 \cdot 4) \cdot 30 = 13 \cdot 30 = 390\ cm^2$

$A'_p = 0.0\ cm^2$

7.3.2 (3): where A'_p is the area of pre or post-tensioned tendons within $A_{c,eff}$

$\rho_{p,eff} = 24.50 \,/\, 390 = 0.06282$

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \cdot \dfrac{f_{ct,eff}}{\rho_{p,eff}} \cdot (1 + \alpha_e \cdot \rho_{p,eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

7.3.4 (1): Eq. 7.9: the difference of the mean strain in the reinforcement and in the concrete

$f_{ct,eff} = f_{ctm} = 0.30 \cdot f_{ck}^{2/3} = 2.565 \approx 2.6\ MPa$

(NDP) 7.3.2 (2): $f_{ct,eff}$ mean value of concrete tensile strength, here no minimum value of $f_{ct,eff} \geq 3.0\ MPa$ is set

$f_{ct,eff} = 2.6 < 3.0\ MPa \;\rightarrow f_{ct,eff} = 3.0\ MPa$

Tab. 3.1: $f_{ctm} = 2.6\ MPa$ for C 25 or $f_{ctm} = 0.30 \cdot f_{ck}^{2/3} = 2.565\ MPa$

$$\epsilon_{sm} - \epsilon_{cm} = \frac{264.06 - 0.4 \cdot \dfrac{2.565}{0.06282} \cdot (1 + 6.354 \cdot 0.06282)}{200000}$$

k_t: factor dependent on the duration of the load, $k_t = 0.4$ for long term loading

$$\epsilon_{sm} - \epsilon_{cm} = 1.2060 \cdot 10^{-3} > 0.6 \cdot \frac{264.06}{200000} = 0.79218 \cdot 10^{-3}$$

$$s_{r,max} = \frac{\phi}{3.6 \cdot \rho_{p,eff}} \leq \frac{\sigma_s \cdot \phi}{3.6 \cdot f_{ct,eff}}$$

(NDP) 7.3.4 (3): Eq. 7.11: $s_{r,max}$ is the maximum crack spacing

$$s_{r,max} = \frac{25}{3.6 \cdot 0.06282} = 110.545\ mm$$

$$s_{r,max} \leq \frac{\sigma_s \cdot \phi}{3.6 \cdot f_{ct,eff}} = 611.25\ mm$$

$w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm}) = 110.545 \cdot 1.2060 \cdot 10^{-3}$

7.3.4 (1): Eq. 7.8: w_k the crack width

$w_k = 0.133 < 0.3\ mm \;\rightarrow$ Check for crack width passed with given reinforcements

Stress limitation

$\sigma_{max,t} = k_3 \cdot f_{yk}$

7.2 (5)

$\sigma_{max,t} = 0.80 \cdot 500\ MPa$

$\sigma_{max,t} = 400\ MPa$

5 Conclusion

This example shows the calculation of crack widths. Various ways of reference calculations are demonstrated, in order to compare the SOFiSTiK results to. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.