Benchmark Example No. 57

Response Spectrum Analysis of a Simply Supported Beam

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VERiFiCATiON
BE57 Response Spectrum Analysis of a Simply Supported Beam

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Front Cover

Volkstheater, Munich   Photo: Florian Schreiber

Overview	
Element Type(s):	
Analysis Type(s):	
Procedure(s):	
Topic(s):	EQKE
Module(s):	DYNA
Input file(s):	response_spectrum_analysis.dat

1 Problem Description

The following example is focused on the results of a response spectrum analysis of a simply supported beam, with the problem description defined by [1]. A simple beam with a rectangular cross section, shown in Fig.1a, is subjected to a vertical movement of its supports according to the acceleration history shown in Fig.1b. The acceleration changes linearly from g to $-g$ within a time period of $2t_d$, and is zero afterwards, with g being the gravitational acceleration.

The response of the system is determined based only on the first eigenmode. Therefore, modal superposition is not carried out in this example. Furthermore, zero damping of the system is assumed.

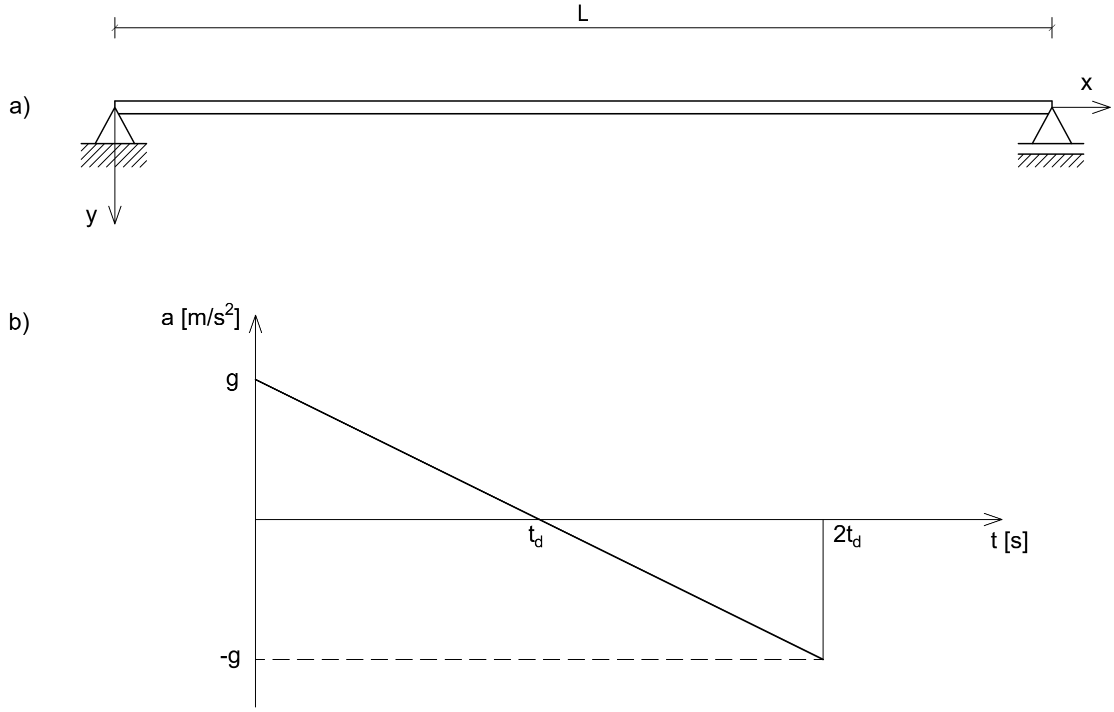

Figure 1: (a) Simply supported beam; (b) Acceleration history

The acceleration of a mass in a SDOF spring-mass system subjected to the base acceleration history from Fig.1b is defined as follows [1], for $t \leq 2t_d$:

$$\ddot{u} = g\left(1 - \cos\omega t + \frac{\sin\omega t}{\omega t_d} - \frac{t}{t_d}\right) \tag{1}$$

and for $t > 2t_d$:

$$\ddot{u} = \ddot{u}_{t=2t_d} \cos\omega(t - 2t_d) + \dot{u}_{t=2t_d} \frac{\sin\omega(t - 2t_d)}{\omega} \tag{2}$$

where ω is the circular eigenfrequency.

2 Reference Solution

In the first step of the response spectrum analysis, the eigenfrequency of the first eigenmode is calculated as follows [1]:

$$f = \frac{\pi}{2l^2}\sqrt{\frac{EI_y}{m}} \tag{3}$$

where l is the length and m is the mass per unit lenght of the beam, and EI_y is the bending stiffness.

Based on the calculated eigenfrequency, the maximum relative displacement of the equivalent SDOF system, $u_{max,0}$, is determined from the corresponding response spectrum [1]. Subsequently, the maximum beam deflection is calculated [1]:

$$u_{max} = \Gamma u_{max,0} \tag{4}$$

The shape function for the first eigenmode $\Phi(x)$ is given by [1]

$$\Phi(x) = \sin\frac{\pi x}{l} \tag{5}$$

, and the modal participation factor Γ is calculated as:

$$\Gamma = \frac{\int_0^l m\Phi(x)dx}{\int_0^l m[\Phi(x)]^2 dx} = \frac{4}{\pi} \tag{6}$$

The bending moment is defined as follows [1]:

$$M_x = -EI_y \frac{\partial^2 u(x)}{\partial x^2} \tag{7}$$

where $u(x)$ is the beam deflection for the first eigenmode

$$u(x) = u_{max} \sin \frac{\pi x}{l} \tag{8}$$

Therefore, the maximum bending moment in the middle of the span is computed as:

$$M_{y,max} = \frac{EI_y \pi^2}{l^2} u_{max} \tag{9}$$

3 Model and Results

Material, geometry and loading properties of the beam model defined in a plane system are summarized in Table 1.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 206842\,MPa$	$l = 6.096\,m$	$g = 10\,m/s^2$
$\rho = 104730\,kg/m^3$	$h = 35.56\,mm$	$t_d = 0.1\,s$
	$b = 3.7026\,mm$	

The response spectrum values are calculated as maximum acceleration values, in the units of g, from the Equations 1 and 2 as a function of the eigenperiod. For the purpose of this example, only the values in the proximity of the system´s first eigenperiod are taken as the input points of the response spectrum. The selected points are listed in Table 2 and also shown on the graph of the response spectrum, which is plotted as a function of the eigenfrequency in Figure 2.

Table 2: Calculated points of the response spectrum

Eigenfrequency [Hz]	Eigenperiod [s]	Max. acceleration [g]
5.00	0.20	2.000000
5.50	0.181818	1.818181
6.00	0.166667	1.666667
6.05	0.165289	1.652893
6.10	0.163934	1.639344
6.15	0.162602	1.626016
6.50	0.1538461	1.538461
7.00	0.142857	1.428571

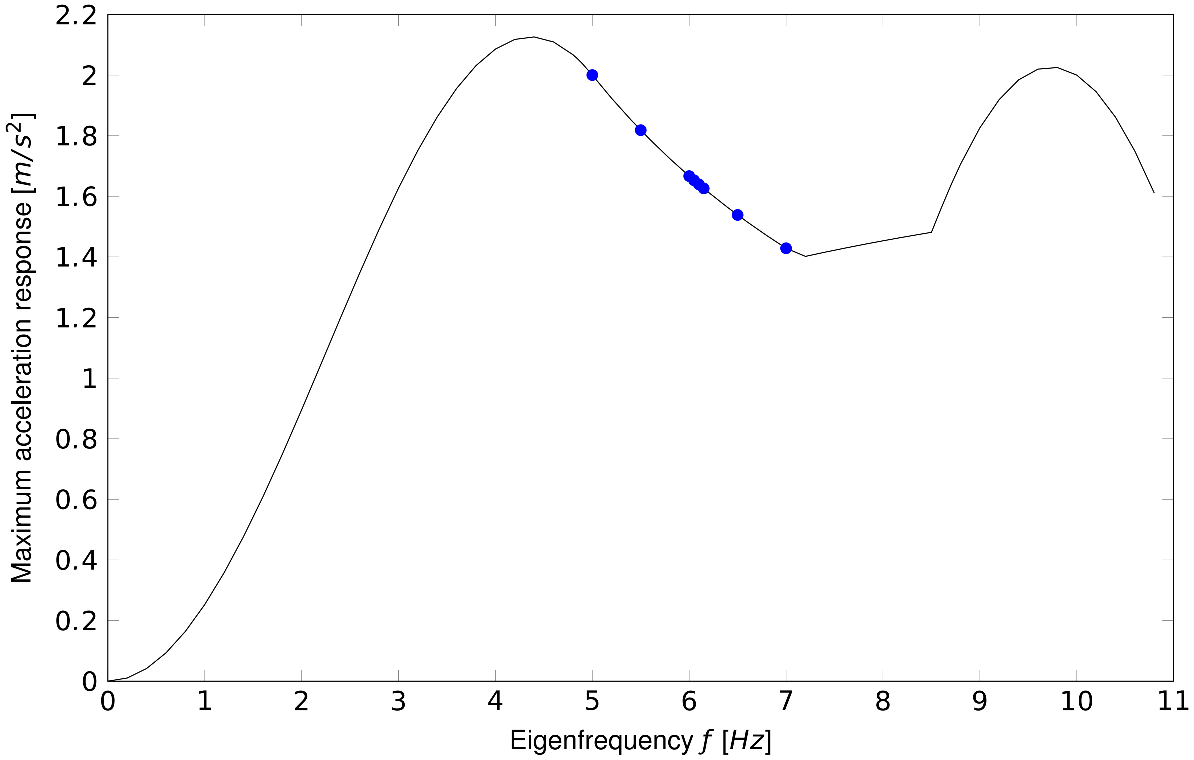

Figure 2: The response spectrum and the selected input points

The calculated values of the eigenfrequency f of the first mode, and the maximum deflection u_{max} and the bending moment $M_{y,max}$ in the middle of the span as a result of the response spectrum analysis, are compared with the reference values in Table 3.

Table 3: Results

	SOF.	Ref. [1]
f $[Hz]$	6.12	6.10
u_{max} $[mm]$	14.15	14.22
$M_{y,max}$ $[kNm]$	108.03	108.41

4 Conclusion

A very good agreement between the reference solution and the numerical results calculated by SOFiSTiK verifies the implementation of the response spectrum analysis.

5 Literature

[1] J.M. Biggs. *Structural Dynamics*. McGraw-Hill, 1964.