SOFiSTiK

Benchmark Example No. 56

Interface element

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VERiFiCATiON
BE56 Interface element

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Front Cover

Volkstheater, Munich   Photo: Florian Schreiber

Overview	
Element Type(s):	C2D
Analysis Type(s):	MNL
Procedure(s):	
Topic(s):	Interface Element
Module(s):	SOFIMSHC, TALPA
Input file(s):	interface_elements.dat

1 Problem Description

The following example is focused on verifying the interface element which can be used to model the contact behaviour in a geotechnical model. In the example according to [1] interface elements are simulating the contact between a long elastic block on a rigid foundation. The block is subjected to pressure p at one vertical side, while being restrained at the other end, and no strain is permitted in the y direction (Figure 1).

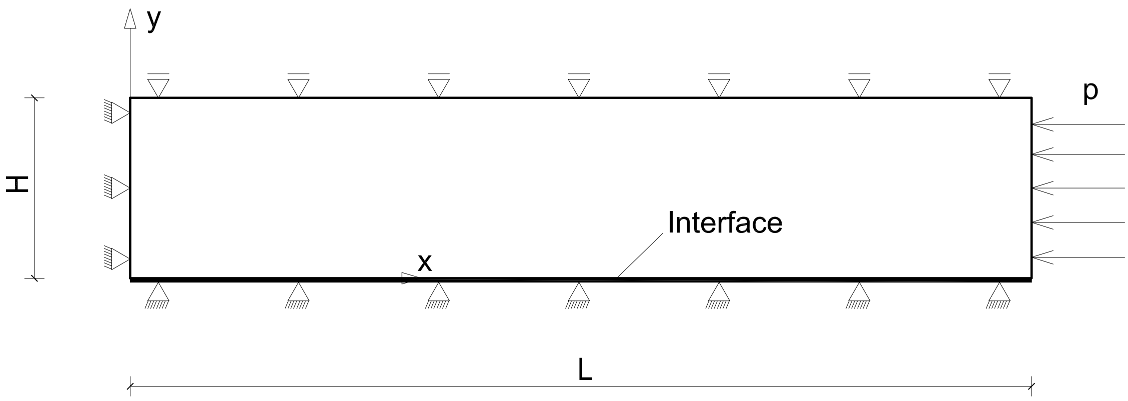

Figure 1: Long elastic block on a rigid foundation

2 Reference Solution

The distribution of shear stress along the interface between $x = 0$ and $x = x_1$, where x_1 is the point at which the shear stress reaches its maximum level, is given analytically by [1]:

$$\tau(x) = \frac{k_s m_v}{\alpha} \cdot \frac{e^{\alpha x} - e^{-\alpha x}}{e^{\alpha x_1} - e^{-\alpha x_1}} \cdot \left[pH - \tau_{max}(L - x_1) \right] \tag{1}$$

where

$m_v = \frac{(1+\nu)(1-2\nu)}{E(1+\nu)}$

$\alpha = \sqrt{\frac{k_s m_v}{H}}$

k_s	shear stiffness parameter,
ν	Poisson's ratio,

E	Elastic modulus,
p	pressure applied at $x = L$,
τ_{max}	maximum shear stress (cohesion)
H	height of the elastic block,
L	length of the elastic block

For the slipping portion of the block, between $x = x_1$ and $x = L$, the shear stress is constant, i.e. $\tau = \tau_{max}$.

The point $x = x_1$ is calculated iteratively by applying the Newton Raphson iterative scheme for the following equation [1, 2]:

$$\frac{e^{\alpha x} + e^{-\alpha x}}{e^{\alpha x_1} - e^{-\alpha x_1}} + \alpha(L - x_1) - \frac{p\alpha H}{\tau_{max}} = 0 \tag{2}$$

3 Model and Results

Material, geometry and loading properties of the model are summarized in the Table 1. To satisfy the required condition of no strain in the y direction the normal stiffness of the interface elements, i.e. the elastic constant normal to the interface surface c_s is defined with a relatively high value. Plane strain conditions are assumed, and nonlinear analysis is performed with loading being increased in increments of 2.5 kPa up to 400 kPa.

Table 1: Model Properties

Material	Geometry	Loading
Elastic block:		Increments of 2.5 kPa
$E = 100\ MPa$, $\nu = 0.0$	$L = 10.0\ m, H = 1.0\ m$	up to 400 kPa
Interface elements:		
$k_s = c_t = 10^4\ kN/m^3$	Thickness of $0.01\ m$	
$\tau_{max} = coh = 30\ kN/m^2$		
$c_s = 10^7\ kN/m^3$		

The shear stress distribution along the interface length is plotted in Figure 2, and verified with respect to the formulas provided in Section 2 for the loading levels of 100, 200, 300 and 400 kPa. Furthermore, the longitudinal displacement distribution at the bottom of the elastic block is compared with the results of the finite element analysis provided by [1] (Figure 3).

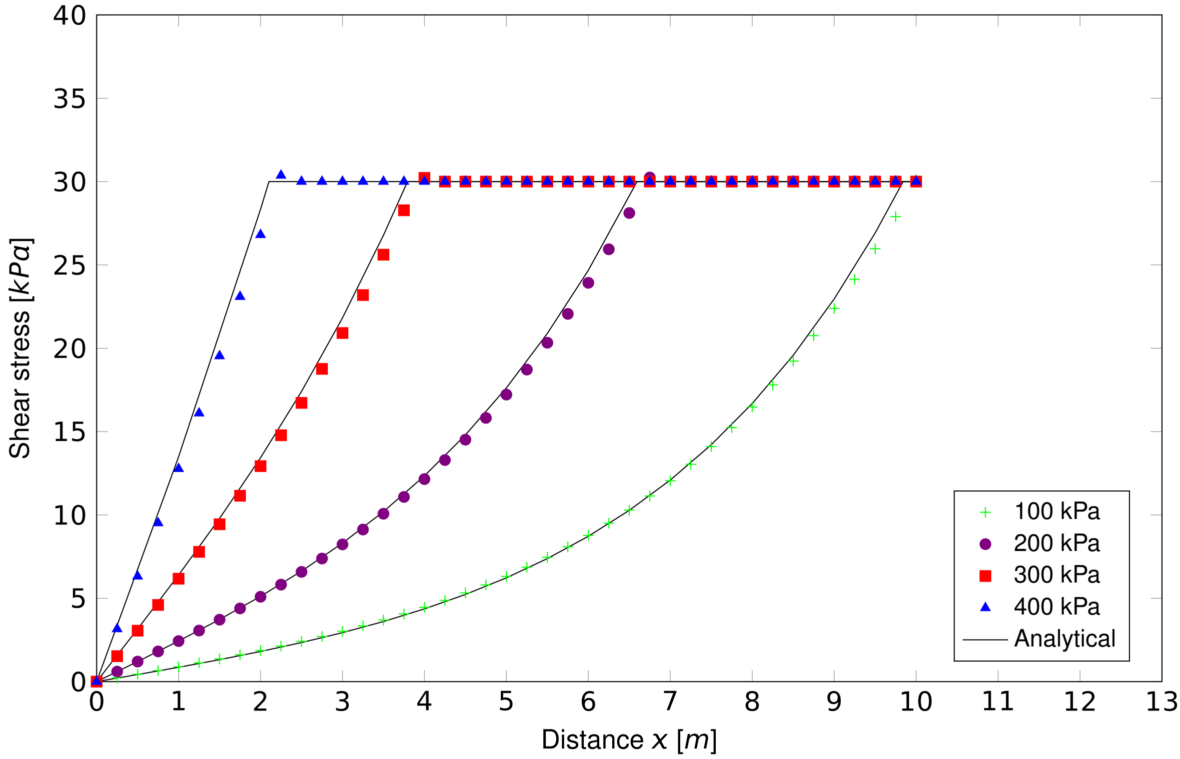

Figure 2: Interface shear stress distribution

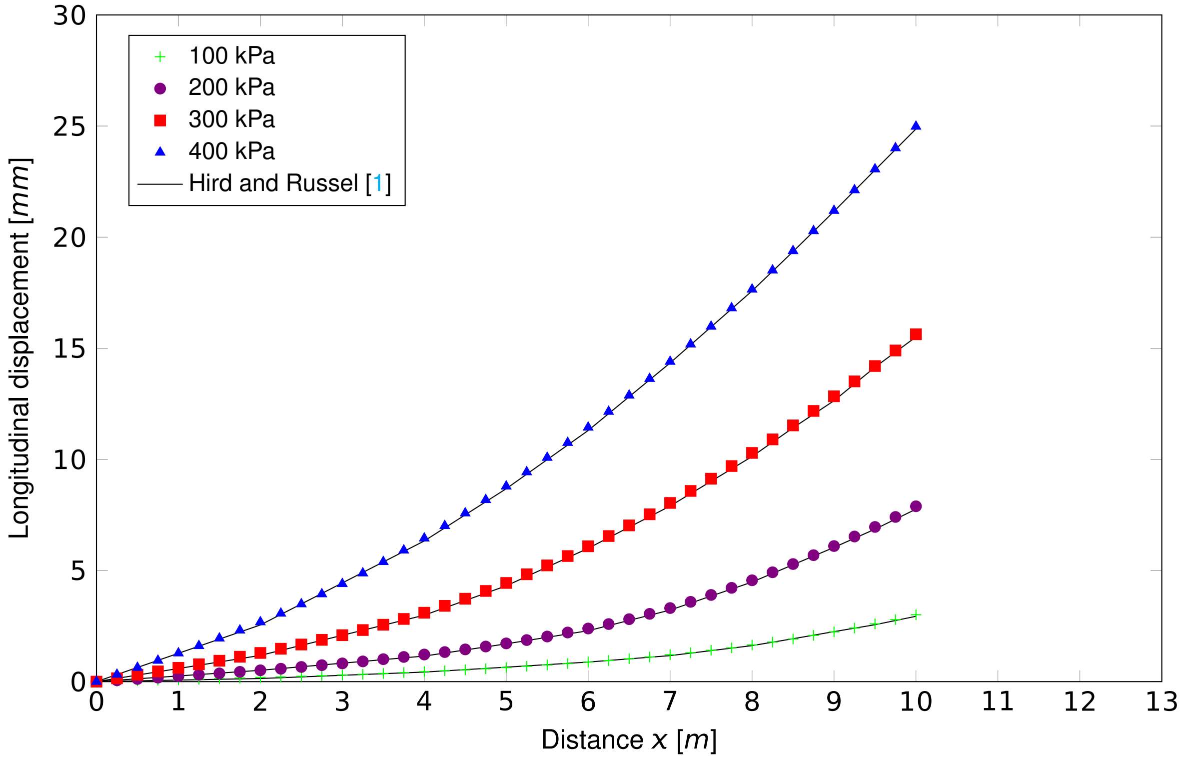

Figure 3: Longitudinal displacement distribution at the bottom of the elastic block

When comparing the numerical results in Figure 2 with the analytical solution a slight difference can be noticed. It should be noted that the analytical solution provided in Section 2 is not exact since it is based on the assumption that the normal stresses are constant along the height of the elastic block [1].

In reality, the normal stress will be higher near the unrestrained upper boundary compared to the lower one, which can be seen in Figure 4 for a distance of 2 m from the restrained vertical face of the elastic block.

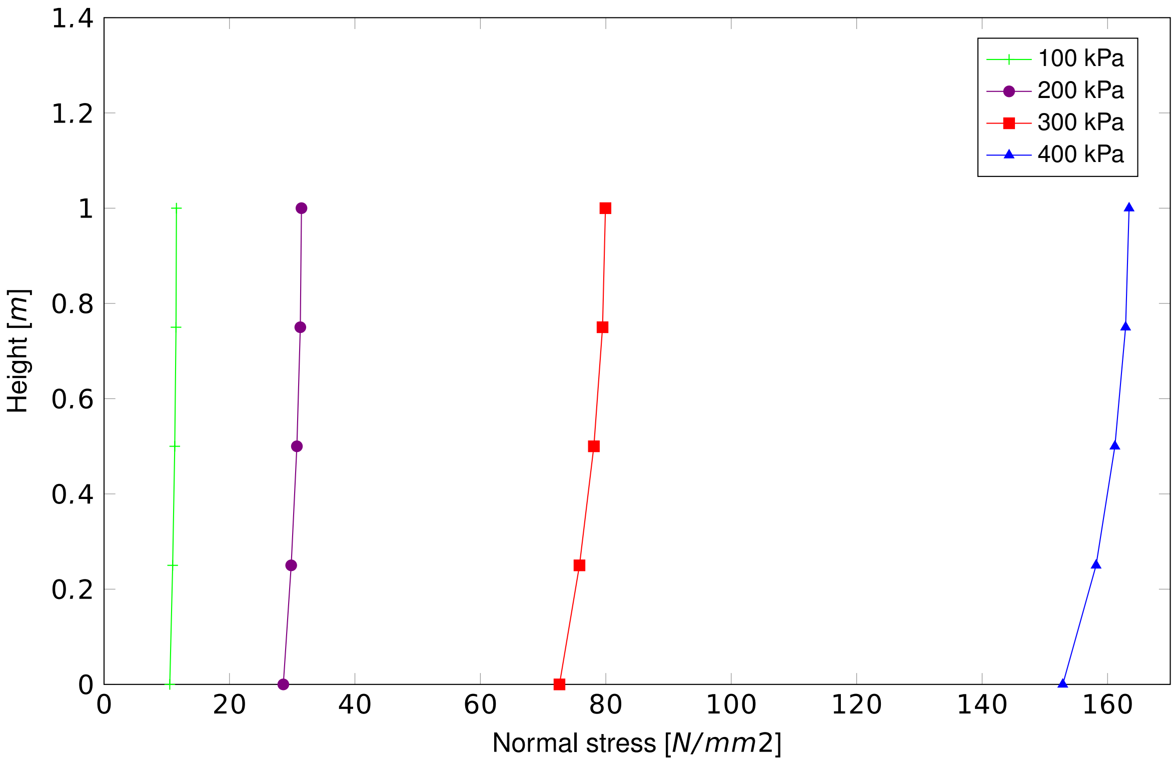

Figure 4: Stress distribution along the height of the block, at $x = 2.0$ [m]

4 Conclusion

A good agreement between the reference solution and the numerical results calculated by SOFiSTiK verifies the implementation of the interface element.

5 Literature

[1] C.C. Hird and D. Russell. "A Benchmark for Soil-Structure interface Elements". In: *Computers and Geotechnics* (1990).
[2] R.C. Barros et al. "A Benchmark for Soil-Structure interface Elements". In: *Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering* (2017).