SOFiSTiK

Benchmark Example No. 18

Three-storey Column under Large Compressive Force and Torsional Moment

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VERiFiCATiON
BE18 Three-storey Column under Large Compressive Force and Torsional Moment

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Front Cover

Volkstheater, Munich   Photo: Florian Schreiber

Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT, GNL
Procedure(s):	
Topic(s):	
Module(s):	ASE, DYNA
Input file(s):	three_storey_column.dat

1 Problem Description

The problem consists of a three-storey column, subjected to a large compressive axial force N and a torsional moment M_t at the middle, as shown in Fig. 1. The rotation and twisting as well as the torsional moments of the structure are determined.

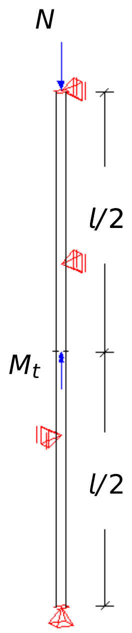

Figure 1: Problem Description

2 Reference Solution

A large axial compressive force is applied to the column of Fig. 1, in combination with a torsional moment at the middle, which can cause warping and potentially buckling of the structure. In order to account for this effect, second order theory has to be utilised. The total torsional moment M_T is given as a sum of the different torsional parts, the primary, secondary and third respectively:

$$\sum M_x = M_T = M_{T1} + M_{T2} + M_{T3}, \tag{1}$$

where

$$M_{T1} = G\, I_T\, \phi', \tag{2}$$

$$M_{T2} = -\, E\, C_M\, \phi''', \tag{3}$$

$$M_{T3} = N\, i_p^2\, \phi', \tag{4}$$

and the warping moment

$$M_\omega = -\, E\, C_M\, \phi''. \tag{5}$$

where G is the shear modulus, I_T the torsional moment of inertia, i_p the polar radius of gyration and EC_M the warping torsion stiffness. Introducing the above into Eq. 1 we have:

$$\left(G\, I_T \,+\, N\, i_p^2\right) \phi' - E\, C_M\, \phi''' \,=\, M_T \,= \sum M_x. \tag{6}$$

3 Model and Results

The properties of the model [1] are defined in Table 1. A standard steel material is used and an I-beam profile for the cross-section. A safety factor $\gamma_M = 1.1$ is used, which according to DIN 18800-2 it is applied both to the yield strength and the stiffness. At the supports the warping is not constrained. The cross-sectional properties given in Table 1 are the values calculated by SOFiSTiK, matching the analytical solution, except from the torsional moment I_T and the warping modulus C_M which are modified to match the values of the reference example. This modification is done only for the sake of comparison and it has to be noted that the reference results [1] are computed with another finite element software, and not with respect to an analytical solution.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$\gamma_M = 1.1$	$b = 180\, mm$	$N = 1712\, kN$
$l = 6\, m$	$h = 400\, mm$	$M_t = 272\, kN\, cm$
$S\, 355$	$t_{web} = 10\, mm$	
	$t_{flange} = 14\, mm$	
	$C_M = 506884\, cm^6$	
	$I_y = 23071.6\, cm^4$	
	$I_z = 1363.9\, cm^4$	
	$I_T = 44.18\, cm^4$	

The results are presented in Table 2 and Fig 2. The value of M_{T3} is not given in the Reference [1], but according to Eq. 4 is computed as $-721\ kNcm$, which matches the calculated value by SOFiSTiK. If we now sum the torsional moment parts, it is observed that Eq. 1 is satisfied and that the total torsional moment at $x = 0$ is $136\ kNcm$.

Table 2: Results

	SOF.	Ref.[1]
$\phi\ [mrad]\ (x = l/2)$	294.4	294
$\phi'\ [mrad/cm]\ (x = 0; x = l)$	1.5096	1.50965
$M_{T1}\ [kN\,cm]\ (x = 0)$	491	491
$M_{T2}\ [kN\,cm]\ (x = 0)$	366	366
$M_{T3}\ [kN\,cm]\ (x = 0)$	-721	-
$M_{T}\ [kN\,cm]\ (x = 0)$	136	136
$M_{\omega}\ [kN\,cm]$	85583	85620

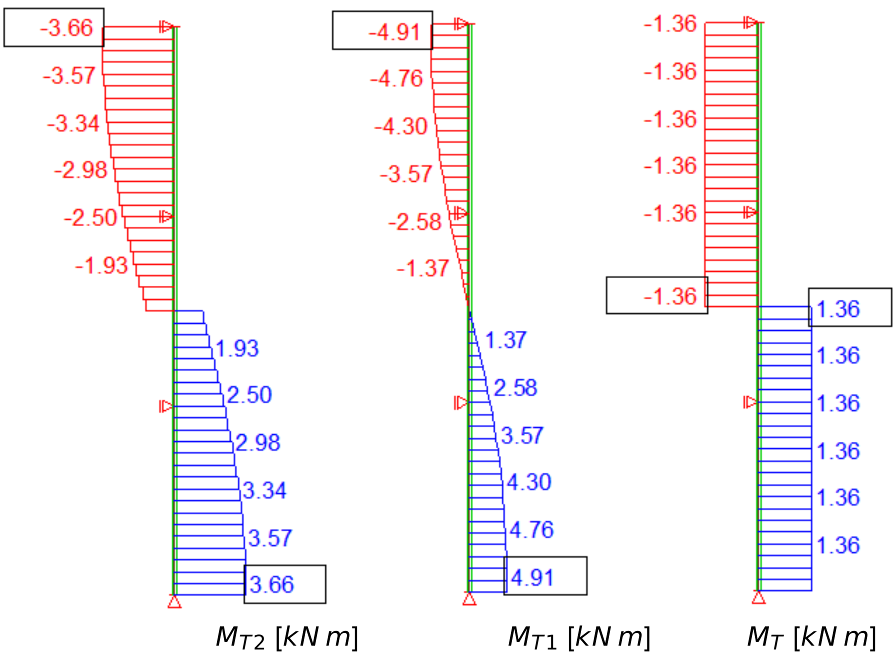

Figure 2: Results $[kN\,m]$

4 Conclusion

This example examines the torsional behaviour of the beam and the different parts involved in the calculation of the total torsional moment. The results are reproduced accurately.

5 Literature

[1] V. Gensichen and G. Lumpe. *Anmerkungen zur linearen und nichtlinearen Torsionstheorie im Stahlbau*. Stahlbau Seminar 2012.