Benchmark Example No. 10

Verification of Beam and Section Types II

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VERiFiCATiON
BE10 Verification of Beam and Section Types II

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SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Volkstheater, Munich   Photo: Florian Schreiber

Overview

Element Type(s): B3D
Analysis Type(s): STAT
Procedure(s):
Topic(s):
Module(s): ASE
Input file(s): cross_sections_ii_FEM.dat, cross_sections_ii_BEM.dat

1 Problem Description

The problem consists of a cantilever beam as shown in Fig. 1. For the first case analysed, a transverse load is applied at the end of the beam. For the second case, a moment is applied around the x axis. The various cross-section types analysed in Benchmark Example 9 are used, in order to test the behaviour of the beam associated with each of the section definitions.

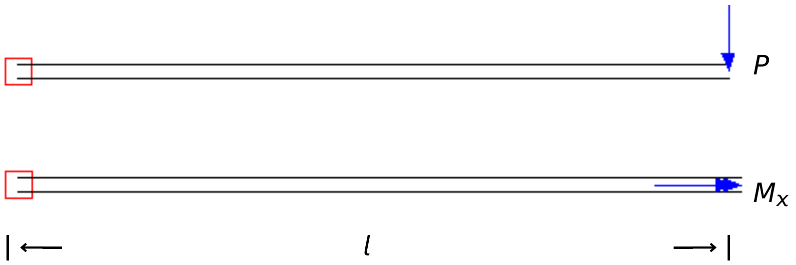

Figure 1: Problem Description

2 Reference Solution

For a Bernoulli beam and a linear elastic material behaviour, the maximum deflection δ_{max} of the cantilever, under the action of a transverse load P, occurs at the tip and is [1]:

$$\delta_{max} = \frac{PL^3}{3EI}, \tag{1}$$

and the rotation ϕ_z

$$\phi_z = \frac{PL^2}{2EI}. \tag{2}$$

For the case of the moment M, applied at the x-axis the angle of twist ϕ_x is [2]:

$$\phi_x = \frac{ML}{GI_T}, \tag{3}$$

where G is the shear modulus, EI the flexural rigidity and I_T the torsional moment.

3 Model and Results

The properties of the model and the cross-sections analysed, are defined in Table 1. For all cross-sections the shear deformation areas A_y and A_z are given equal to zero, in order to consider a Bernoulli beam formulation which doesn't account for shear deformations.

Table 1: Model Properties

Material Properties	Cross-sectional Properties	Loading
$E = 30\,MPa$	$L = 1\,m$	$P = 1\,kN$
$\nu = 0.3$	$h = 100\,mm$	$M = 1\,kNm$
	$t = 10\,mm$	
	$b = 100\,mm$	
	$D = 100\,mm$	

Table 2: Results Case 1

	$u_y\ [m]$		$\lvert e_r \rvert$	$\phi_z\ [mrad]$		$\lvert e_r \rvert$
Type	SOF.	Ref.	$[\%]$	SOF.	Ref.	$[\%]$
square -srec	1.333	1.333	0.00	2.000	2.000	0.00
rectangular -srec	1333.333	1333.333	0.00	2000.000	2000.000	0.00
circul -scit	2.264	2.264	0.00	3.395	3.395	0.00
circul -tube	2.264	2.264	0.00	3.395	3.395	0.00
pipe -scit	3.834	3.834	0.00	5.751	5.751	0.00
pipe -tube	3.834	3.834	0.00	5.751	5.751	0.00
Tbeam -poly	6.173	6.173	0.00	9.259	9.259	0.00
Tbeam -plat	6.126	6.078	0.80	9.189	9.116	0.80
Ibeam -poly	2.473	2.473	0.00	3.709	3.709	0.00
Ibeam -plat	2.386	2.377	0.36	3.578	3.566	0.36
Ibeam -weld	2.482	2.473	0.37	3.723	3.709	0.37
square box -poly	2.258	2.258	0.00	3.388	3.388	0.00
square box -plat	2.286	2.279	0.31	3.429	3.419	0.31
square box open -plat	2.286	2.279	0.31	3.429	3.419	0.31
rectang. box -poly	1.236	1.236	0.00	1.855	1.855	0.00
rectang. box -plat	1.247	1.250	0.21	1.871	1.874	0.21
C-beam -poly	0.485	0.485	0.00	0.727	0.727	0.00

Table 2: (continued)

		$u_y\ [m]$	$\lvert e_r \rvert$		$\phi_z\ [mrad]$	$\lvert e_r \rvert$
Type	SOF.	Ref.	[%]	SOF.	Ref.	[%]
C-beam -plat	0.486	0.486	0.07	0.729	0.728	0.07
L-beam -poly	6.173	6.173	0.00	9.259	9.259	0.00
L-beam -weld	6.199	6.173	0.42	9.298	9.259	0.42
L-beam -plat	6.221	6.193	0.44	9.331	9.290	0.44

The cross-sections types are modelled in various ways in AQUA as shown in Benchmark Example 9. The results are presented in Table 2 for the case of the transverse load P and in Table 3 for the case of the moment M. For the non-circular cross sections modelled with -POLY, both the results calculated with the boundary element method (BEM) and the finite element method (FEM) are presented in Table 3. It should be noted, that the calculated angle of twist for the square and rectangular box cross section modelled with -POLY (FEM), denoted with a star in Table 3, corresponds to a relatively coarse default finite element mesh. For the investigated box sections with relatively thin walls, a better approximation in regard to the reference values can be obtained by implementing a finer element mesh.

Table 3: Results Case 2

		$\phi_x\ [mrad]$	$\lvert e_r \rvert$
Type	SOF.	Ref.	[%]
square -srec	6.165	6.190	0.41
rectangular -srec	2774.886	2768.903	0.22
circul -scit	8.828	8.828	0.00
circul -tube	8.828	8.828	0.00
pipe -scit	14.952	14.952	0.00
pipe -tube	14.952	14.952	0.00
Tbeam -poly (BEM)	1343.070	1368.421	1.85
Tbeam -poly (FEM)	1359.899		0.62
Tbeam -plat	1333.333	1333.333	0.00
Ibeam -poly (BEM)	910.375	928.571	1.96
Ibeam -poly (FEM)	895.95		3.51
Ibeam -plat	896.552	896.552	0.00
Ibeam -weld	928.571	928.571	0.00
square box -poly (BEM)	11.227	11.888	5.56
square box -poly (FEM - default mesh)	10.877⋆		8.51
square box -poly (FEM - finer mesh: HDIV 2 [mm])	11.221		5.61

Table 3: (continued)

	ϕ_x $[mrad]$		$\|e_r\|$
Type	SOF.	Ref.	[%]
square box -plat	11.696	11.888	1.62
square box open -plat	723.428	722.222	0.17
rectang. box -poly (BEM)	3.991	4.149	3.83
rectang. box -poly (FEM - default mesh)	3.901⋆		5.98
rectang. box -poly (FEM - finer mesh: HDIV 2 [mm])	3.998		3.64
rectang. box -plat	4.113	4.149	0.89
C-beam -poly (BEM)	652.274	684.211	4.67
C-beam -poly (FEM)	679.255		0.72
C-beam -plat	684.210	684.211	0.00
L-beam -poly (BEM)	1363.46	1368.421	0.36
L-beam -poly (FEM)	1385.289	1368.421	1.23
L-beam -weld	1368.421	1368.421	0.00
L-beam -plat	1368.421	1368.421	0.00

From the above results, and with respect to the results of Benchmark Example 9, we can see that the differences are a direct influence of the calculations of the properties of the cross-sections according to their definition in AQUA, and are not associated to the beam formulation. This can also be verified, if instead of, e.g. the reference value for $I_{y_{REF}}$, the calculated value is used $I_{y_{CALC}}$ in Eq. 1 . Then the error is eliminated for all the cross-sections types.

4 Conclusion

This example presents the influence of the cross-sections types, for the case of a simple cantilever beam. It has been shown that the behaviour of the beam is accurately captured.

5 Literature

[1] K. Holschemacher. *Entwurfs- und Berechnungstafeln für Bauingenieure*. 3rd. Bauwerk, 2007.
[2] C. Petersen. *Stahlbau*. 2nd. Vieweg, 1990.