Benchmark Example No. 1

Creep and Shrinkage Calculation using the Model Code 2010

SOFiSTiK | 2022

VERiFiCATiON
DCE-MC1 Creep and Shrinkage Calculation using the Model Code 2010

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Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview	
Design Code Family(s):	MC
Design Code(s):	MC 2010
Module(s):	AQB, CSM
Input file(s):	creep_shrinkage_mc10.dat

1 Problem Description

The problem consists of a simply supported beam with a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The total creep and shrinkage is calculated.
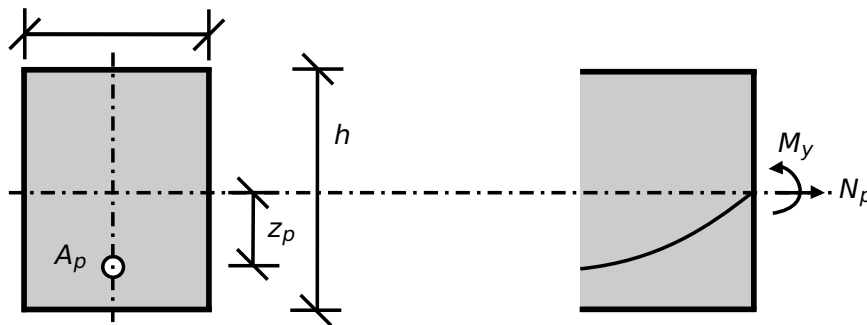

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the calculation of creep and shrinkage on a prestressed concrete cross-section, subject to bending and prestress force. The content of this problem is covered by the following parts of fib Model Code 2010 [1]:

- Creep and Shrinkage (Section 5.1.9.4)

- Temperature effects (Section 5.1.10)

3 Model and Results

Benchmark 17 is here extended for the case of creep and shrinkage developing on a prestressed concrete simply supported beam. In benchmark 18 the calculation was made using DIN EN 1992-1-1:2004 design code. This example will explain the calculation for the case of creep and shrinkage using fib Model Code 2010 [1] The analysed system can be seen in Fig. 2, with properties as defined in Table 1. Further information about the tendon geometry and prestressing can be found in benchmark 17. The beam consists of a rectangular cross-section and is prestressed and loaded with its own weight. A calculation of the creep and shrinkage is performed with respect to fib Model Code 2010 [1].

Table 1: Model Properties

Material Properties	Geometric Properties	Time
C 35/45	$h = 100.0 \, cm$	$t_0 = 28 \, days$
Y 1770	$b = 100.0 \, cm$	$t_s = 0 \, days$

Table 1: (continued)

Material Properties	Geometric Properties	Time
$RH = 80\ \%$	$L = 20.0\ m$	$t = 36500\ days$
	$A_p = 28.5\ cm^2$	
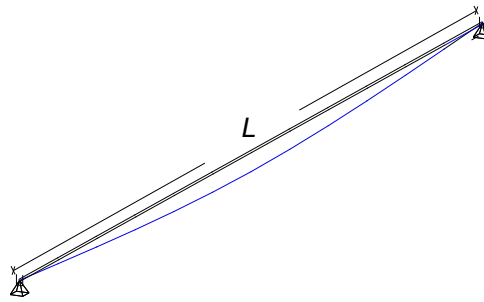
Figure 2: Simply Supported Beam

Table 2: Results

Result	AQB	CSM+AQB	Ref.
ϵ_{cs}	$-27.8 \cdot 10^{-5}$	$-27.8 \cdot 10^{-5}$	$-27.82 \cdot 10^{-5}$
$\phi_{bc}(t, t_0)$	1.57	1.561	1.563

Note: The results from SOFiSTiK are rounded for output.

4   Design Process

Design with respect to fib Model Code 2010 [1]

Material:

Concrete: $C\ 35/45$

$E_{cm} = 35000\ N/mm^2$

$f_{ck} = 35\ N/mm^2$

$f_{cm} = 43\ N/mm^2$

Prestressing Steel: $Y\ 1770$

$E_p = 195000\ N/mm^2$

$f_{pk} = 1770\ N/mm^2$

CALCULATION OF TOTAL SHRINKAGE AND SWELLING at $x = 10.0\ m$ midspan:

$t_0 = 28$ days

$t_s = 0$ days

$t = 36500$ days

$\epsilon_{cs}(t, t_s) = \epsilon_{cbs}(t) + \epsilon_{cds}(t, t_s)$

Calculating the basic shrinkage:

$\epsilon_{cbs}(t) = \epsilon_{cbs0}(f_{cm}) \cdot \beta_{bs}(t)$

$\epsilon(f_{cm}) = -\alpha_{bs} \cdot \left(\dfrac{0.1 \cdot f_{cm}}{6 + 0.1 \cdot f_{cm}} \right)^{2.5} \cdot 10^{-6}$

$\alpha_{bs} = 700$ for N class of cement

$\epsilon(f_{cm}) = -700 \cdot \left(\dfrac{0.1 \cdot 43}{6 + 0.1 \cdot 43} \right)^{2.5} \cdot 10^{-6}$

$\epsilon(f_{cm}) = -700 \cdot (4.3/10.3)^{2.5} \cdot 10^{-6}$

$\epsilon(f_{cm}) = -7.8827 \cdot 10^{-5}$

The development of drying shrinkage strain in time strongly depends on $\beta_{ds}(t, t_s)$ factor. SOFiSTiK accounts not only for the age at start of drying t_s but also for the influence of the age of prestressing t_0. Therefore, the calculation of factor β_{ds} reads:

$\beta_{bs}(t) = 1 - exp(-0.2 \cdot \sqrt{t}) - \left(1 - exp(-0.2 \cdot \sqrt{t_0})\right)$

$\beta_{bs}(t) = 1 - exp(-0.2 \cdot \sqrt{36500}) - \left(1 - exp(-0.2 \cdot \sqrt{28})\right)$

$\beta_{bs}(t) = 1 - exp(-38.2099) - (1 - exp(-1.0583))$

$\beta_{bs}(t) = 0.347$

5.1: Concrete

5.1.7.2:   Modulus of elasticity for $C\ 35/45$

5.1.4:   Mean value of compressive strength f_{ck}. See the eq. (5.1-1)

5.3: Prestressing Steel

E_p for wires

f_{pk} Characteristic tensile strength of prestressing steel

t_0 age at first loading
t_s concrete age at the beginning of shrinkage or swelling
t age of concrete at the moment considered

5.1.9.4.4: Eq. 5.1-75: $\epsilon_{cs}(t, t_s)$ total shrinkage or swelling strains

5.1.9.4.4: Eq. 5.1-76: $\epsilon_{cbs}(t)$ is the basic shrinkage

5.1.9.4.4: Eq. 5.1-78: $\epsilon_{cds0}(f_{cm})$ is the notional shrinkage coefficient

α_{bs} is a coefficient, dependent on the type of cement (see table 5.1-12)

The basic shrinkage is calculated:

$$\epsilon_{cbs}(t) = \epsilon_{cbs0}(f_{cm}) \cdot \beta_{bs}(t)$$

5.1.9.4.4: Eq: 5.1-76

$$\epsilon_{cbs}(t) = -7.8827 \cdot 10^{-5} \cdot 0.347$$

$$\epsilon_{cbs}(t) = -0.0002735269 = -2.735 \cdot 10^{-5}$$

Calculating the drying shrinkage:

5.1.9.4.4: Eq. 5.1-77

$$\epsilon_{cds}(t, t_s) = \epsilon_{cds0}(f_{cm}) \cdot \beta_{RH}(RH) \cdot \beta_{ds}(t - t_s)$$

The drying shrinkage is calculated $\epsilon_{cds}(t, t_s)$ by means of the notional drying shrinkage coefficient $\epsilon_{cds0}(f_{cm})$, the coefficient β_{RH}, taking into account the effect of the ambient relative humidity, and the function $\beta_{ds}(t - t_s)$ describing the time development:

5.1.9.4.4: Eq. 5.1-80

$$\epsilon_{cds0} = [(220 + 110 \cdot \alpha_{ds1}) \cdot exp(-\alpha_{ds2} \cdot f_{cm})] \cdot 10^{-6}$$

See table 5.1-12

Coefficients (α_{dsi}) are depending on the type of cement.

For normal class type of cement:
$\alpha_{ds1} = 4$
$\alpha_{ds2} = 0.012$

$$\epsilon_{cds0}(f_{cm}) = [(220 + 110 \cdot 4) \cdot exp(-0.012 \cdot f_{cm})] \cdot 10^{-6}$$

$$\epsilon_{cds0}(f_{cm}) = [660 \cdot exp(-0.516)] \cdot 10^{-6}$$

$$\epsilon_{cds0}(f_{cm}) = 39.39 \cdot 10^{-5}$$

5.1.9.4.4: Eq. 5.1-81

$$\beta_{RH} = \begin{cases} -1.55 \cdot \left[1 - \left(\dfrac{RH}{100}\right)^3\right], & for \ 40 \leq RH < 99\% \cdot \beta_{s1} \\ 0.25, & for \ RH \geq 99\% \cdot \beta_{s1} \end{cases}$$

$$\beta_{s1} = \left(\frac{35}{f_{cm}}\right)^{0.1} \leq 1.0$$

5.1.9.4.4: Eq. 5.1-83

$$\beta_{s1} = \left(\frac{35}{43}\right)^{0.1} = 0.9796 \leq 1.0$$

$$99\% \cdot \beta_{s1} = 99 \cdot 0.9796 = 96.98$$

$$\beta_{RH} = -1.55 \cdot \left[1 - \left(\frac{80}{100}\right)^3\right] = -0.7564$$

SOFiSTiK accounts not only for the age at start of drying t_s but also for the influence of the age of prestressing, so the time development function reads:

5.1.9.4.4: Eq. 5.1-82

$$\beta_{ds}(t - t_s) = \sqrt{\frac{t - t_s}{0.035 \cdot h^2 + (t - t_s)}} - \sqrt{\frac{t_0 - t_s}{0.035 \cdot h^2 + (t_0 - t_s)}}$$

$$\beta_{ds}(t - t_s) = \sqrt{\frac{36500}{0.035 \cdot 500^2 + 36500}} - \sqrt{\frac{28}{0.035 \cdot 500^2 + 28}}$$

$$\beta_{ds}(t - t_s) = 0.8981 - 0.05669 = 0.8416$$

The drying shrinkage is calculated:
$$\epsilon_{cds}(t, t_s) = \epsilon_{cds0}(f_{cm}) \cdot \beta_{RH} \cdot \beta_{ds}(t - t_s)$$

$\epsilon_{cds}(t, t_s) = 39.39 \cdot 10^{-5} \cdot (-0.7564) \cdot 0.8416$

$\epsilon_{cds}(t, t_s) = -25.08 \cdot 10^{-5}$

The total shrinkage or swelling strain is calculated:

$\epsilon_{cs}(t, t_s) = \epsilon_{cbs}(t) + \epsilon_{cds}(t, t_s)$

$\epsilon_{cs}(t, t_s) = (-2.735 + (-25.08)) \cdot 10^{-5} = -27.82 \cdot 10^{-5}$

CALCULATION OF TOTAL CREEP at x=10.0 m midspan:

The creep coefficient:

$\phi(t, t_0) = \phi_{bc}(t, t_0) + \phi_{dc}(t, t_0)$

5.1.9.4.3(b): Eq. 5.1-63: $\phi(t, t_0)$ is the creep coefficient; $\phi_{bc}(t, t_0)$ is the basic creep coefficient according to eq. 5.1-64; $\phi_{dc}(t, t_0)$ is the drying creep coefficient according to eq. 5.1-67

Calculating the basic creep:

$\phi_{bc}(t, t_0) = \beta(f_{cm}) \cdot \beta_{bc}(t, t_0)$

with: $\beta_{bc}(f_{cm}) = \dfrac{1.8}{(f_{cm})^{0.7}} = \dfrac{1.8}{(43)^{0.7}} = 0.12937$

and the time development function:

$\beta_{bc}(t, t_0) = ln\left[\left(\dfrac{30}{t_{0,adj}} + 0.035\right)^2 \cdot (t - t_0) + 1\right]$

5.1.9.4.3(b): Eq. 5.1-66: $\beta_{bc}(t, t_0)$ is the time development function

$t_{0,adj} = t_{0,T} \cdot \left[\dfrac{9}{2 + t_{0,T}^{1.2}} + 1\right]^{\alpha} \geq 0.5 \text{ days}$

$t_{0,adj}$ is the modified age at loading t_0

$t_T = \sum_{i=1}^{n} \Delta t_i \cdot exp\left[13.65 - \dfrac{4000}{273 + T(\Delta t_i)}\right]$

5.1.10.2: Eq. 5.1-85; t_T the adjusted concrete age, during the effect of elevated or reduced temperatures on the maturity of concrete

$t_{0,T} = \sum_{i=1}^{n} 28 \cdot exp\left[13.65 - \dfrac{4000}{273 + 20}\right] = 27.947 \text{ days}$

$\alpha = 0$ for N class cement

$t_{0,adj} = 27.947 \cdot \left[\dfrac{9}{2 + 27.947^{1.2}} + 1\right]^0 = 27.947 \geq 0.5 \text{ days}$

$\beta_{bc}(t, t_0) = ln\left[\left(\dfrac{30}{27.947} + 0.035\right)^2 \cdot 36472 + 1\right]$

$\beta_{bc}(t, t_0) = 10.71$

The basic creep coefficient:

$\phi_{bc}(t, t_0) = 0.12937 \cdot 10.71 = 1.385$

Calculating the drying creep:

The drying coefficient may be estimated from:

$\phi_{dc}(t, t_0) = \beta_{dc} f_{cm} \cdot \beta(RH) \cdot \beta_{dc}(t_0) \cdot \beta_{dc}(t, t_0)$

5.1.9.4.3b: Eq. 5.1-67; $\phi_{dc}(t, t_0)$ is the drying creep coefficient

with:

$\beta_{dc}(f_{cm}) = \dfrac{412}{(f_{cm})^{1.4}} = 2.1283$

5.1.9.4.3: Eq.5.1-68

5.1.9.4.3(b): Eq. 5.1-69

$$\beta(RH) = \frac{1 - \dfrac{RH}{100}}{\sqrt[3]{0.1 \cdot \dfrac{h}{100}}} = \frac{1 - \dfrac{80}{100}}{\sqrt[3]{0.1 \cdot \dfrac{500}{100}}} = 0.251$$

5.1.9.4.3: Eq.5.1-70

$$\beta_{dc}(t_0) = \frac{1}{0.1 + t_{0,adj}^{0.2}} = \frac{1}{0.1 + 27.947} = 0.4886$$

5.1.9.4.3(b): Eq. 5.1-71a; $\beta_{dc}(t, t_0)$ the development of drying creep with time

$$\beta_{dc}(t, t_0) = \left[\frac{t - t_0}{\beta_h + (t - t_0)} \right]^{\gamma(t_0)}$$

with:

5.1.9.4.3: Eq. 5.1-71b

$$\gamma(t_0) = \frac{1}{2 + \dfrac{3.5}{\sqrt{t_{0,adj}}}} = \frac{1}{2.962} = 0.3376$$

5.1.9.4.3: Eq. 5.1-71d

$$\alpha_{fcm} = \sqrt{\frac{35}{fcm}} = \sqrt{\frac{35}{43}} = 0.9021$$

5.1.9.4.3: Eq. 5.1-71c

$$\beta_h = 1.5 \cdot h + 250 \cdot \alpha_{fcm} \leq 1500 \cdot \alpha_{fcm}$$
$$\beta_h = 1.5 \cdot 500 + 250 \cdot 0.9021 = 975.548 \leq 1352.15$$

5.1.9.4.3: Eq. 5.1-71a

$$\beta_{dc}(t, t_0) = \left[\frac{36500 - 28}{975.548 + (36500 - 28)} \right]^{0.3376} = 0,9911$$

The drying creep coefficient:

$$\phi_{bc}(t, t_0) = 2.1283 \cdot 0.251 \cdot 0.4886 \cdot 0.9911 = 0.2597$$

The total creep coefficient:

$$\phi(t, t_0) = \phi_{bc}(t, t_0) + \phi_{dc}(t, t_0)$$
$$\phi(t, t_0) = 1.385 + 0.2587 = 1.64$$

According to Model Code 2010 [1], the creep value is related to the tangent Young's modulus E_c, where E_c being defined as $1.05 \cdot E_{cm}$. To account for this, SOFiSTiK adopts this scaling for the computed creep coefficient (in SOFiSTiK, all computations are consistently based on E_{cm}.

$$\phi(t, t_0) = 1.64/1.05 = 1.56$$

5 Conclusion

This example shows the calculation of the creep and shrinkage using fib Model Code 2010 [1]. It has been shown that the results are in very good agreement with the reference solution.

6 Literature

[1] fib Model Code 2010. *fib Model Code for Concrete Structures 2010*. International Federation for Structural Concrete (fib). 2010.