Benchmark Example No. 8

Design of a Rectangular CS for Shear and Axial Force

SOFiSTiK | 2022

VERiFiCATiON
DCE-EN8 Design of a Rectangular CS for Shear and Axial Force

VERiFiCATiON Manual, Service Pack 2022-12 Build 74

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Front Cover

Arnulfsteg, Munich Photo: Hans Gössing

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	rectangular_shear_axial.dat

1 Problem Description

The problem consists of a rectangular section, symmetrically reinforced for bending, as shown in Fig. 1. The cross-section is designed for a shear force V_{Ed} and a compressive force N_{Ed} and and the required reinforcement is determined.

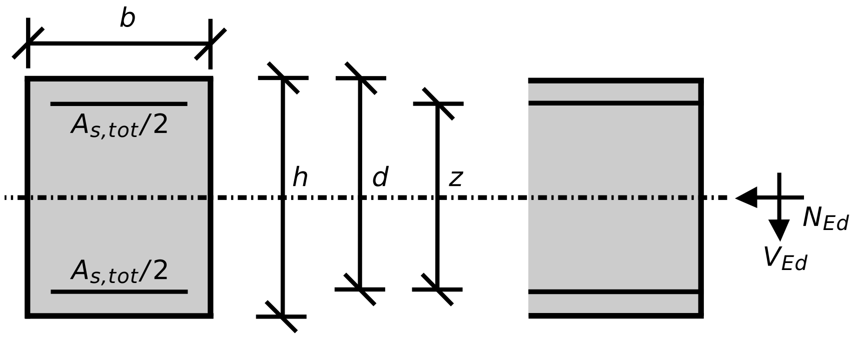

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force and axial force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)
- Reinforcement (Section 9.2.2)

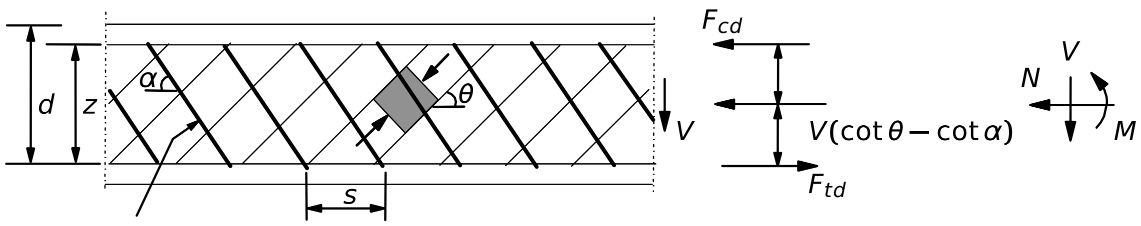

Figure 2: Shear Reinforced Members

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and defined in DIN EN 1992-1-1:2004 [1].

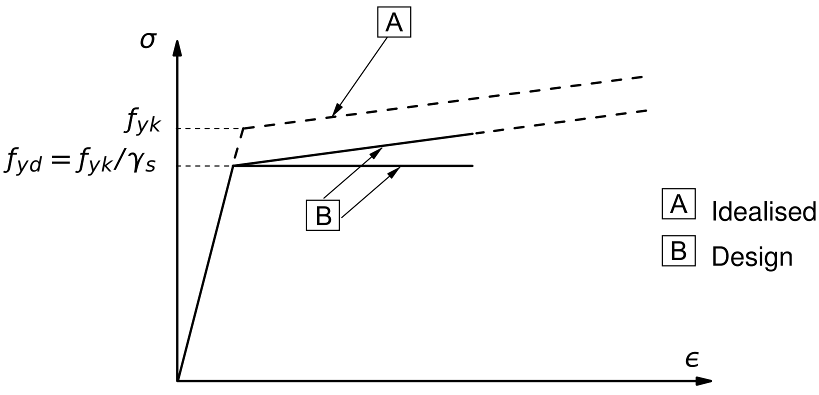

Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under a shear force of $343.25\ kN$ and a compressive axial force of $500.0\ kN$. The reference calculation steps are presented below and the results are given in Table 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\ 30/37$	$h = 50.0\ cm$	$V_{Ed} = 343.25\ kN$
$B\ 500A$	$b = 30\ cm$	$N_{Ed} = 500.0\ kN$
	$d = 45.0\ cm$	
	$A_{s,tot} = 38.67\ cm^2$	
	$c_{V,l} = 3.6\ cm$	

Table 2: Results

	SOF.	Ref.
$A_{sw}\ /\ s\ \ [cm^2/m]$	11.27	11.27
$V_{Rd,c}\ \ [kN]$	132.71	132.71
$\cot\theta$	1.82	1.82

4 Design Process[1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

$f_{ck} = 30\ MPa$

$f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 30/1.5 = 17.0\ MPa$

Tab. 3.1: Strength for concrete

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$

$f_{yd} = f_{yk} / \gamma_s = 500/1.15 = 434.78\ MPa$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

3.2.7: (2), Fig. 3.8

Design Load: $V_{Ed} = 343.25\ kN$

$N_{Ed} = -500.0\ kN$

$z = \max\left\{d - c_{V,l} - 30\ mm;\ d - 2\ c_{V,l}\right\}$

(NDP) 6.2.3 (1): Inner lever arm z

$z = \max\{384;\ 378\} = 384\ mm$

$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$

(NDP) 6.2.2 (1): Eq. 6.2a: Design value for shear resistance $V_{Rd,c}$ for members not requiring design shear reinforcement

with a minimum of

$\left(\nu_{min} + 0.12 \cdot \sigma_{cp}\right) \cdot b_w \cdot d$

(NDP) 6.2.2 (1): Eq. 6.2b

$C_{Rd,c} = 0.15/\gamma_c = 0.1$

(NDP) 6.2.2 (1): $C_{Rd,c} = 0.15/\gamma_c$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{450}} = 1.6667 < 2.0$$

$$\rho_1 = \frac{A_{s,tot}/2}{b_w d} = 0.01432 < 0.02$$

$V_{Rd,c,min} = \left(\nu_{min} + 0.12 \cdot \sigma_{cp}\right) \cdot b_w \cdot d$

$\nu_{min} = (0.0525/\gamma_c) \cdot k^{3/2} \cdot f_{ck}^{\ 1/2} = 0.41249$

(NDP) 6.2.2 (1): Eq. 6.3aDE: ν_{min} for $d \leq 600\ mm$

$V_{Rd,c,min} = 109.68\ kN$

$\sigma_{cp} = N_{Ed}\ /\ A_c < 0.2 \cdot f_{cd}$

(NDP) 6.2.2 (1): Eq. 6.2

$\sigma_{cp} > 0$ for compression

$\sigma_{cp} = -500 \cdot 10^{-3}\ /\ 0.15 \cdot 10^6 = -3.3333\ N/mm^2 < 3.4$

$V_{Rd,c} = \left[0.1 \cdot 1.6667 \cdot (1.432 \cdot 30)^{1/3} + 0.112 \cdot 3.3333\right] \cdot 0.3 \cdot 0.45 = 132.71\ kN$

$V_{Ed} > V_{Rd,c} \rightarrow$ shear reinforcement is required

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

(NDP) 6.2.3 (2): Eq. 6.7aDE

$$1.0 \leq \cot\theta \leq \frac{1.2 + 1.4\ \sigma_{cd} / f_{cd}}{1 - V_{Rd,cc} / V_{Ed}} \leq 3.0$$

(NDP) 6.2.3 (2): Eq. 6.7bDE
$c = 0.5$

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2 \frac{\sigma_{cd}}{f_{cd}}\right) \cdot b_w \cdot z$$

(NDP) 6.2.3 (2): $\sigma_{cd} = N_{Ed} / A_c$

$$\sigma_{cp} = N_{Ed} / A_c$$

$$\sigma_{cd} = -500 \cdot 10^{-3} / 0.15 \cdot 10^6 = -3.3333\ N/mm^2$$

6.7DE: $\sigma_{cd} > 0$ for compression

$$V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 30^{\ 1/3} \cdot \left(1 - 1.2 \frac{3.3333}{17.0}\right) \cdot 0.3 \cdot 0.384$$

$$V_{Rd,cc} = 65.6948\ kN$$

(NDP) 6.2.3 (2): The angle θ should be limited by Eq. 6.7DE

$$\cot\theta = \frac{1.2 + 1.4 \cdot 3.3333 / 17.0}{1 - 65.6948 / 343.25} = 1.823$$

6.2.3 (3): Eq. 6.8
$f_{ywd} = f_{yk}/\gamma_s = 435\ MPa$

$$A_{sw,requ} / s = V_{Ed} / (f_{ywd} \cdot z \cdot \cot\theta) = 11.27\ cm^2/m$$

(NDP) 6.2.3 (3): Eq. 6.9
Maximum shear force $V_{Rd,max}$ occurs for $\theta = 45°$: $\cot\theta = \tan\theta = 1$
(NDP) $\nu_1 = 0.75 \cdot \nu_2 = 0.75$, $\nu_2 = 1$ for $\leq C50/60$

$$V_{Rd,max} = b_w \cdot z \cdot \nu_1 \cdot f_{cd} / (\cot\theta + \tan\theta)$$

$$V_{Rd,max} = 0.3 \cdot 0.384 \cdot 0.75 \cdot 17 / (1 + 1) = 734.4\ kN$$

5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under shear with compressive axial force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.