Benchmark Example No. 6

Design of a Rectangular CS for Shear Force

SOFiSTiK | 2022

VERiFiCATiON
DCE-EN6 Design of a Rectangular CS for Shear Force

VERiFiCATiON Manual, Service Pack 2022-12 Build 74

SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview	
Design Code Family(s):	EN, DIN
Design Code(s):	DIN EN 1992-1-1, EN 1992-1-1
Module(s):	AQB
Input file(s):	rectangular_shear.dat

1 Problem Description

The problem consists of a rectangular section, symmetrically reinforced for bending, as shown in Fig. 1. The cross-section is designed for shear force V_{Ed} and the required shear reinforcement is determined.

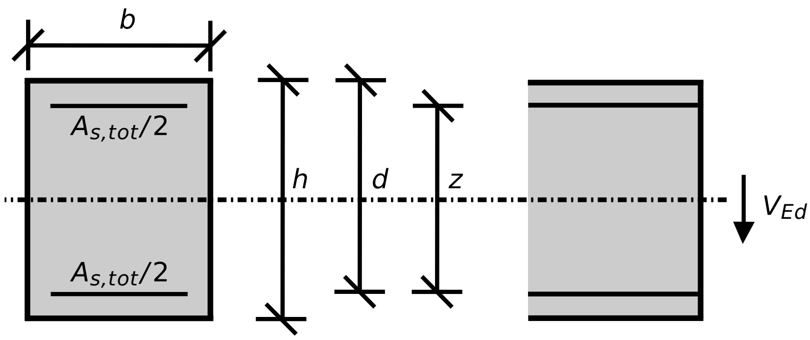


Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)
- Reinforcement (Section 9.2.2)

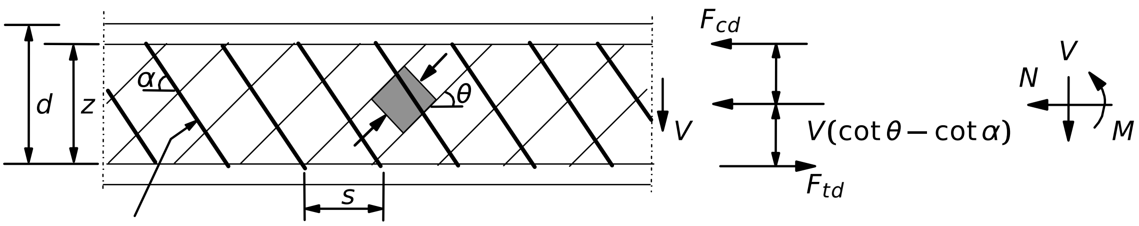


Figure 2: Shear Reinforced Members

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

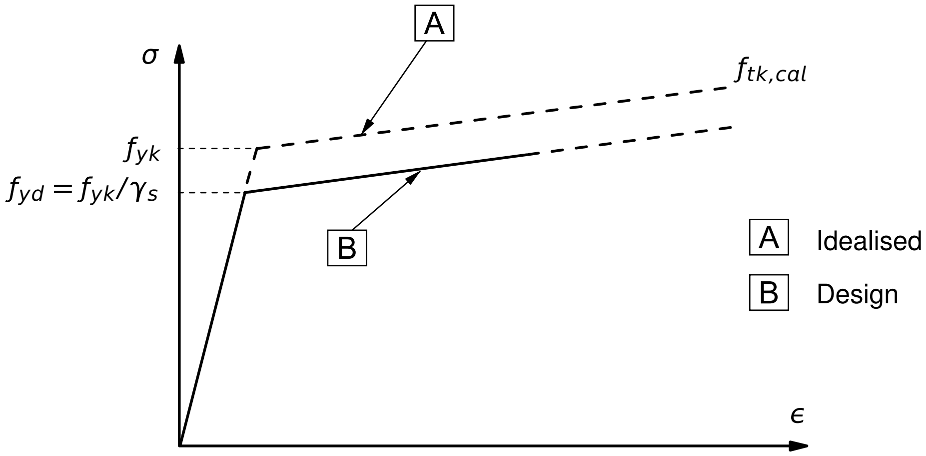


Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The rectangular section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under shear force of $343.25\ kN$. The reference calculation steps are presented below and the results are given in Table 2. Then, the same section is designed with repsect to EN 1992-1-1:2004 [3]. The same angle $\theta (= 1.60)$ is chosen, as calculated with respect to DIN EN 1992-1-1:2004, in order to compare the results. If no θ value is input, then the calculation starts with the upper limit $\cot\theta = 2.5$ and through an optimization process the right angle is selected. In this case, the reinforcement is determined with $\cot\theta = 2.5$, giving a shear reinforcement of $7.80\ cm^2/m$. Also in order to demonstrate that the correct value of $V_{Rd,max} = 734.4\ kN$ (reference value) with repsect to DIN EN 1992-1-1:2004 is calculated in SOFiSTiK, we input a design shear force of 734.3 delivering a shear reinforcement, but when a value of 734.4 is input then AQB gives the warning of 'no shear design possible' showing that the maximum shear resistance is exceeded.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\ 30/37$	$h = 50.0\ cm$	$V_{Ed} = 343.25\ kN$
$B\ 500A$	$b = 30\ cm$	
	$d = 45.0\ cm$	
	$A_{s,tot} = 38.67\ cm^2$	

Table 2: Results

$A_{s,tot}[cm^2/m]$	Design Code	SOF.	Ref.
	DIN EN [1]	12.84	12.84
	EN [3]	12.18	12.18

4 Design Process[1]

Material:

Concrete: $\gamma_c = 1.50$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

Steel: $\gamma_s = 1.15$

$f_{ck} = 30\ MPa$

Tab. 3.1: Strength for concrete

$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 30/1.5 = 17.0\ MPa$

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

3.2.7: (2), Fig. 3.8

Design Load: $V_{Ed} = 343.25\ kN$

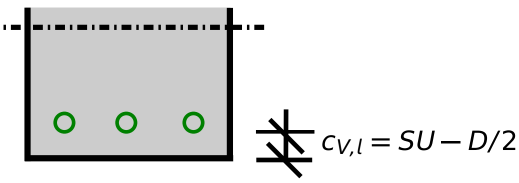
Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

$z = \max \{d - c_{V,l} - 30\ mm;\ d - 2\ c_{V,l}\}$

(NDP) 6.2.3 (1): Inner lever arm z

$c_{V,l} = SU - D/2 = 50 - 28/2 = 36\ mm$

$z = \max \{450 - 36 - 30\ mm;\ 450 - 2 \cdot 36\}$



$z = \max \{384;\ 378\} = 384\ mm$

$$1.0 \leq \cot\theta \leq \frac{1.2 + 1.4\ \sigma_{cd}\ /\ f_{cd}}{1 - V_{Rd,cc}\ /\ V_{Ed}} \leq 3.0$$

(NDP) 6.2.3 (2): Eq. 6.7aDE

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2\frac{\sigma_{cd}}{f_{cd}}\right) \cdot b_w \cdot z$$

(NDP) Eq. 6.2.3 (2): 6.7bDE
$c = 0.5$

$$V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 30^{\ 1/3} \cdot (1 - 0) \cdot 0.3 \cdot 0.384$$

(NDP) 6.2.3 (2): $\sigma_{cd} = N_{Ed}\ /\ A_c$

$$V_{Rd,cc} = 0.08591\ MN = 85.91 kN$$

$$\cot\theta = \frac{1.2 + 0}{1 - 85.91\ /\ 343.25} = 1.60$$

(NDP) 6.2.3 (2): The angle θ should be limited by Eq. 6.7DE

$$A_{sw,requ}\ /\ s = V_{Ed}\ /\ (f_{ywd} \cdot z \cdot \cot\theta) = 12.84\ cm^2/m$$

6.2.3 (3): Eq. 6.8
$f_{ywd} = f_{yk}/\gamma_s = 435\ MPa$

Calculating the max. shear force $V_{Rd,max}$ in general that the concrete can bear $\rightarrow \cot\theta = \tan\theta = 1.0$.

$$V_{Rd,max} = b_w \cdot z \cdot \nu_1 \cdot f_{cd}\ /\ (\cot\theta + \tan\theta)$$

(NDP) 6.2.3 (3): Eq. 6.9
Maximum shear force $V_{Rd,max}$ occurs for $\theta = 45°$: $\cot\theta = \tan\theta = 1$
$\nu_1 = 0.75 \cdot \nu_2 = 0.75$, $\nu_2 = 1$ for $\leq C50/60$

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

$$V_{Rd,max} = 0.3 \cdot 0.384 \cdot 0.75 \cdot 17 \,/\, (1 + 1) = 734.4 \; kN$$

Design with respect to EN 1992-1-1:2004 [3]: [3]

6.2.3 (1): Inner lever arm z

$$z = 0.9 \cdot d$$

$$z = 0.9 \cdot 450 = 405 \; mm$$

6.2.3 (2): Eq. 6.7N
The angle θ should be limited by Eq. 6.7N

$1.0 \leq \cot\theta \leq 2.5 \;\rightarrow\; \cot\theta = 1.60$ (choose for comparison)

$$A_{sw,requ} \,/\, s = V_{Ed} \,/\, (f_{ywd} \cdot z \cdot \cot\theta) = 12.18 \; cm^2/m$$

[3] The sections mentioned in the margins refer to EN 1992-1-1:2004 [3], unless otherwise specified.

5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular cross-section under shear force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] *EN 1992-1-1: Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2004.