Benchmark Example No. 4

Design of a Rectangular CS for Bending and Axial Force

SOFiSTiK | 2022

VERiFiCATiON
DCE-EN4 Design of a Rectangular CS for Bending and Axial Force

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Front Cover

Arnulfsteg, Munich Photo: Hans Gössing

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	rectangular_bending_axial.dat

1 Problem Description

The problem consists of a rectangular section, as shown in Fig. 1. The cross-section is designed for an ultimate moment M_{Ed} and a compressive force N_{Ed} and the required reinforcement is determined.

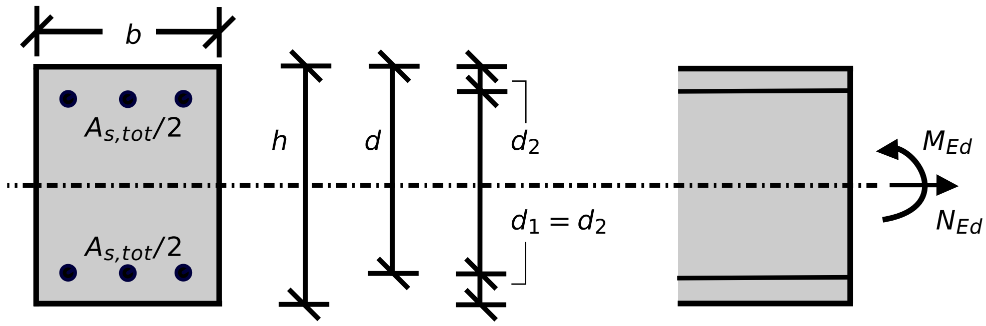

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to bending with axial force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Basic assumptions for section design (Section 6.1)
- Reinforcement (Section 9.3.1.1, 9.2.1.1)

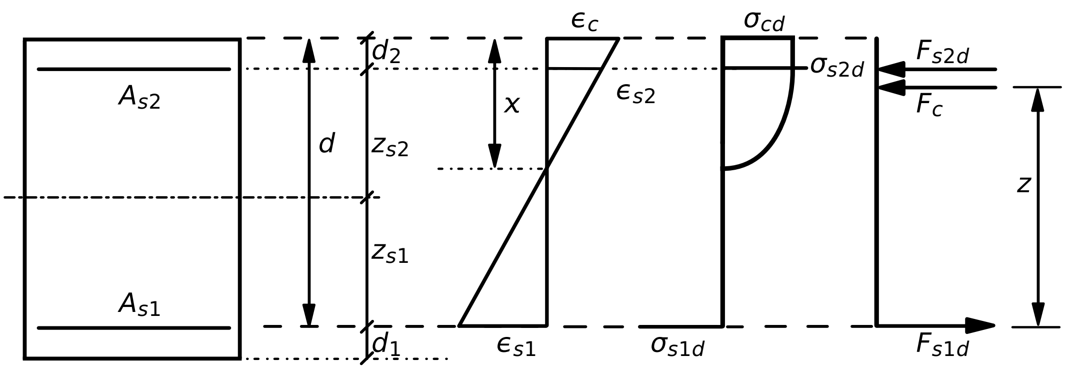

Figure 2: Stress and Strain Distributions in the Design of Doubly Reinforced Cross-sections

In doubly reinforced rectangular beams, the conditions in the cross-section at the ultimate limit state, are assumed to be as shown in Fig. 2. The design stress-strain diagram for reinforcing steel considered in this example, consists of an horizontal top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

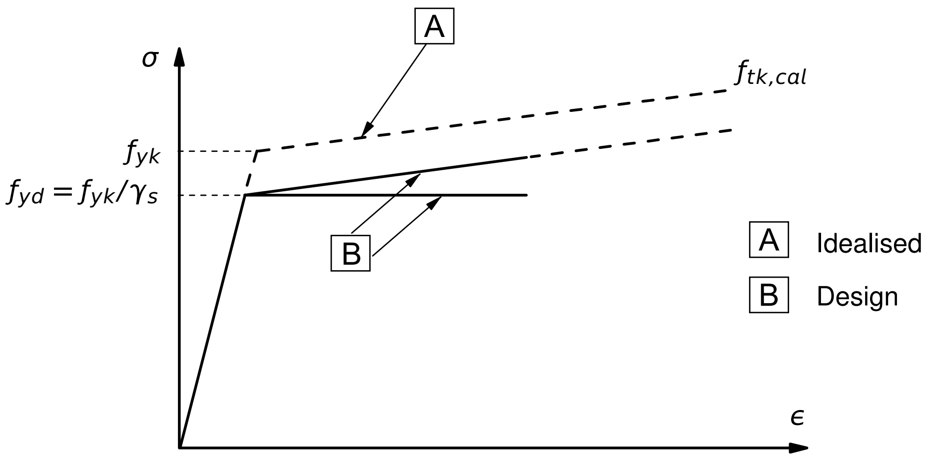

Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The rectangular cross- section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], to carry an ultimate moment of 382 kNm with an axial compressive force of 1785 kN. The calculation steps with a commonly used design method [3] [4] are presented below and the results are given in Table 2. Here, it has to be mentioned that the standard methods employed in order to calculate the reinforcement are approximate, and therefore deviations often occur.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\,30/37$	$h = 50.0\,cm$	$M_{Ed} = 382\,kNm$
$B\,500A$	$d = 45.0\,cm$	$N_{Ed} = -1785\,kN$
	$d_1 = d_2 = 5.0\,cm$	
	$b = 30\,cm$	

Table 2: Results

	SOF.	Interaction Diagram [3]
$A_{s,tot}$ $[cm^2/m]$	35.03	35.19

4 Design Process[1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

Concrete: $\gamma_c = 1.50$
Steel: $\gamma_s = 1.15$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

$f_{ck} = 30\ MPa$
$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 30/1.5 = 17.0\ MPa$

Tab. 3.1: Strength for concrete

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$
$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$
3.2.7: (2), Fig. 3.8

Design Load:

$N_{Ed} = -1785\ kN$
$M_{Ed} = 382\ kNm$

$$\frac{e_d}{h} = \left|\frac{M_{Ed}}{N_{Ed} \cdot h}\right| = \left|\frac{382}{-1785 \cdot 0.50}\right| = 0.428 < 3.5$$

→ Axial force dominant → Design with respect to $\mu - \nu$ interaction diagram is suggested

Design with respect to Interaction diagram for Bending with axial force for rectangular cross-sections:

$$\mu_{Ed} = \frac{M_{Ed}}{b \cdot h^2 \cdot f_{cd}} = \frac{382 \cdot 10^{-3}}{0.30 \cdot 0.50^2 \cdot 17.0} = 0.30$$

$$\nu_{Ed} = \frac{N_{Ed}}{b \cdot h^2 \cdot f_{cd}} = \frac{-1785 \cdot 10^{-3}}{0.30 \cdot 0.50 \cdot 17.0} = -0.70$$

Tab. 9.6 [3]: $\mu - \nu$ Interaction diagram for concrete $C12/15 - C50/60$ - Rectangular cross-section with double symmetric reinforcement.

from design chart for $d_1/h = 0.05/0.5 = 0.10$:

$\omega_{tot} = 0.60$

$$A_{s,tot} = \omega_{tot} \cdot \frac{b \cdot h}{f_{yd}/f_{cd}} = 35.19\ cm^2$$

$$A_{s1} = A_{s2} = \frac{A_{s,tot}}{2} = 17.6\ cm^2$$

[1]The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.
[2]The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under bending with axial force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] K. Holschemacher, T. Müller, and F. Lobisch. *Bemessungshilfsmittel für Betonbauteile nach Eurocode 2 Bauingenieure*. 3rd. Ernst & Sohn, 2012.

[4] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.