Benchmark Example No. 30

Steel column with a class 4 cross-section

SOFiSTiK | 2022

VERiFiCATiON
DCE-EN30 Steel column with a class 4 cross-section

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Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview	
Design Code Family(s):	EN
Design Code(s):	EN 1993-1-1
Module(s):	BDK, AQB, AQUA
Input file(s):	scl_4_sig_neff.dat, scl_4_iterative.dat

1 Problem Description

The problem consists of a simply supported beam with a box cross-section shown in Fig. 1. The design element should be verified against uniform compression as shown in Fig. 2.

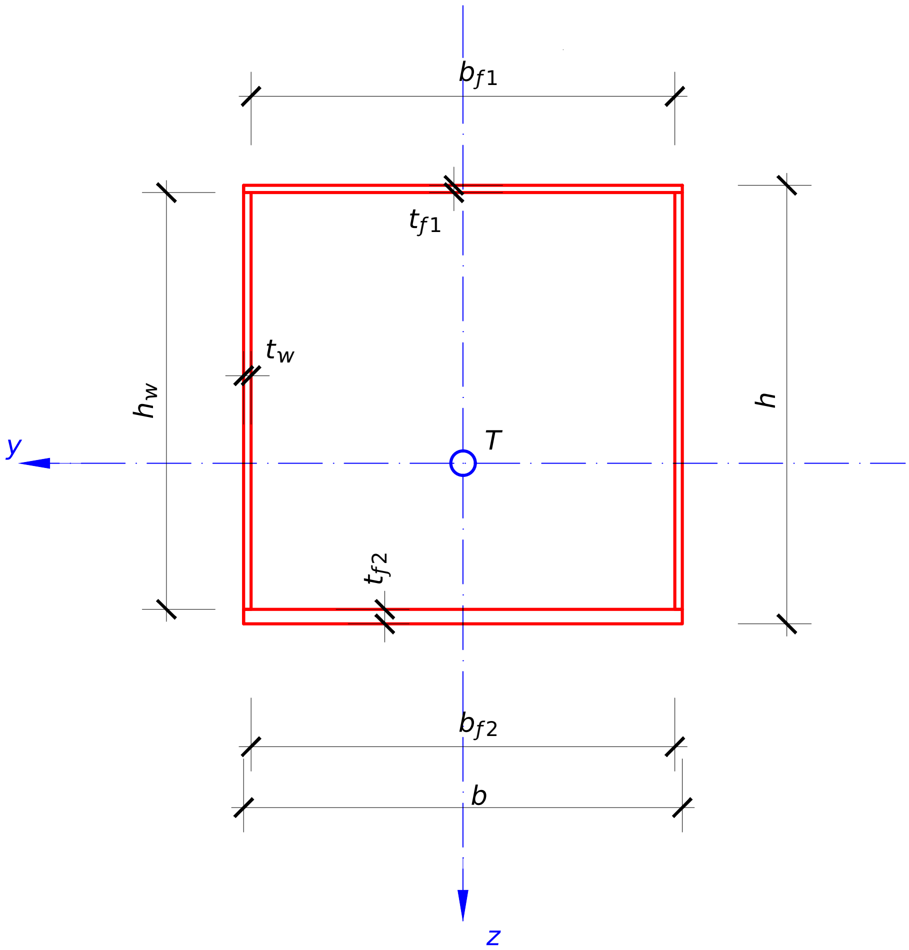

Figure 1: Geometry of box cross-section

This benchmark example is used to verify and compare the SOFiSTiK results with the ECCS reference example [1].

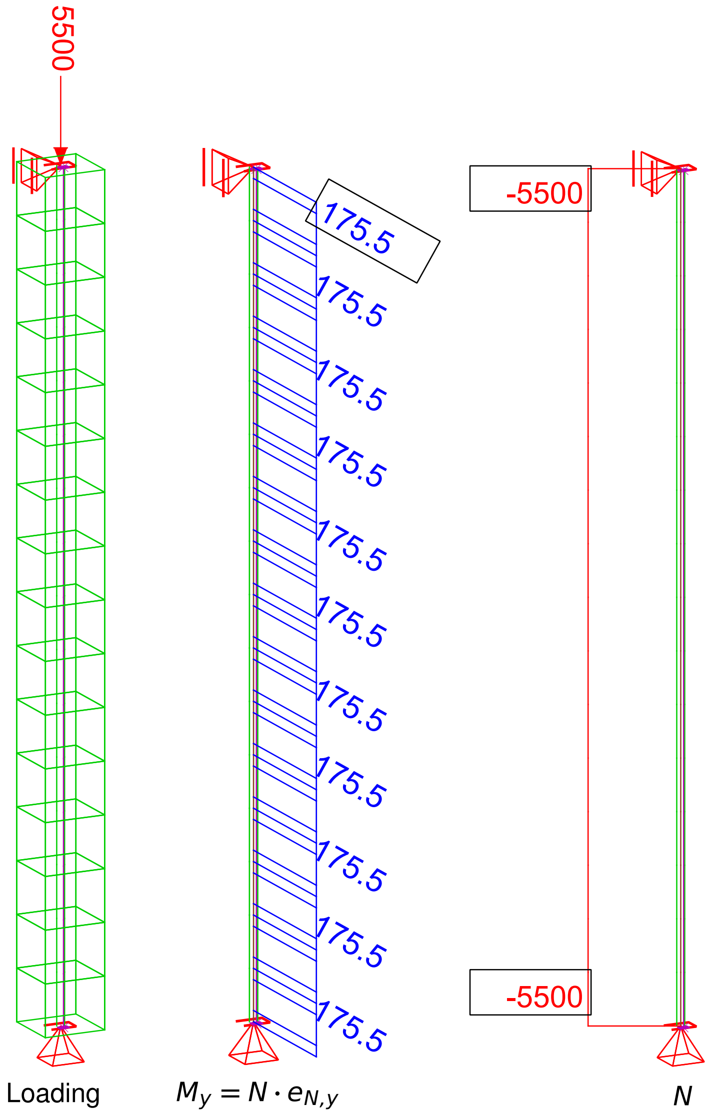

Figure 2: Model, loadings and the internal forces M_y, N

2 Reference Solution

This example is concerned with the cross-section and buckling resistance of steel members. It deals with the spatial behavior of the beam and the occurrence of lateral torsional buckling as a potential mode of failure. The content of this problem is covered by the following parts of EN 1993-1-1:2005 [2]:

- Structural steel (Section 3.2)
- Classification of cross-sections (Section 5.5)
- Buckling resistance of members (Section 6.3)
- Method 2: Interaction factors k_{ij} for interaction formula in 6.3.3(4) (Annex B)

and parts of EN 1993-1-5:2006 [3]

- Effective cross section (Section 4.3)

3 Model and Results

Table 1: Model Properties

Material Properties	Cross-Section Properties	Geometric Properties	Loading
$S\ 275$	$h = 600\ mm$	$H = 4.0\ m$	$N = 5500\ kN$
$E = 210000\ N/mm^2$	$b = 600\ mm$		
$f_y = 275\ N/mm^2$	$t_{f_1} = 10\ mm$		
$\nu = 0.3$	$t_{f_2} = 20\ mm$		
$G = 81000\ N/mm^2$	$t_w = 10\ mm$		
$\gamma_{M0} = 1.0$	$A = 29400\ mm^2$		
$\gamma_{M1} = 1.0$	$I_y = 174800.0\ cm^4$		
	$I_z = 153200.0\ cm^4$		

Table 2: Results

	SOF. (Iterative)	SOF. (SIG NEFF)	Ref. [1][1]
A_{eff} [$cm2$]	275.4	250.2	247.78
$I_{y,eff}$ [cm^4]	1556000	152400	154000.0
$e_{N,y}$ [mm]	38.03	28.20	30.1
$b_{f1,eff}$ [mm]	131.1 ($\rho = 0.77$)	151.3	159.5
$b_{w2,eff}$ [mm]	100.8 ($\rho = 0.82$)	143.24	151.3
$b_{w4,eff}$ [mm]	100.8 ($\rho = 0.82$)	143.24	151.3
$Tot_{utilisation}$	-	0.94	0.95
χ	1.0	1.0	1.0
$\overline{\lambda}_y$	0.183	0.174	0.173
$\overline{\lambda}_z$	0.195	0.186	0.185
k_{yy}	1.080	1.084	1.084
k_{zy}	0.864	0.867	0.867
$nm - y$	0.934	0.952	0.973
$nm - z$	0.893	0.921	0.940

[2]The buckling resistance check has been calculated using hand calculation (SOFiSTiK).

4 Design Process[3]

Design Loads

$N_{Ed} = 5500\ kN$

1. CROSS-SECTION RESISTANCE

STEP 1: Cross-Section class check

$b_{f1} = b_{f2} = b - 2 \cdot t_w = 600 - 2 \cdot 10 = 580\ mm$

$h_w = h - t_{f1} - t_{f2} = 600 - 10 - 20 = 570\ mm$

Upper flange (compression):

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.924$$

$$\frac{b_{f1}}{t_{f1}} = \frac{580}{10} = 58.0 > 42 \cdot \varepsilon = 42 \cdot 0.924 = 38.8 \text{ (Class 4)}$$

Lower flange (compression):

$$\frac{b_{f2}}{t_{f2}} = \frac{580}{20} = 29.0 < 42 \cdot \varepsilon = 38.8 \text{ (Class 3)}$$

Class 3 - but it also fulfils requirements for Class 1 $(33 \cdot \varepsilon)$

Web (compression):

Cross-section classification, EN 1993-1-1, Table 5.2

$$\frac{h_w}{t_w} = \frac{570}{10} = 57.0 > 42 \cdot \varepsilon = 38.8 \text{ (Class 4)}$$

The cross-section is classified as Class 4.

STEP 2: Calculating the effective properties under uniform axial compression

[3]The sections mentioned in the margins refer to DIN EN 1993-1-1:2005 [4] unless otherwise specified.

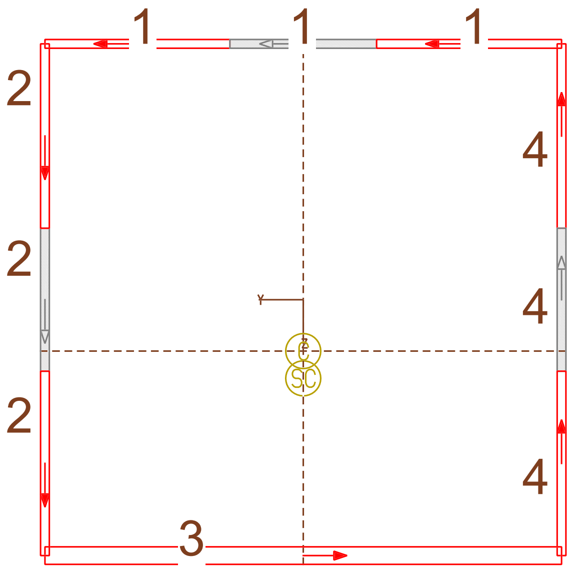

Figure 3: Effective area for uniform compression

Determination of the characteristics of the gross cross section

$$S_y = b \cdot t_{f1} \cdot \left(h - \frac{t_{f1} + t_{f1}}{2}\right) + 2 \cdot h_w \cdot t_w \cdot \left(\frac{h_w + t_{f2}}{2}\right)$$

$$S_y = \frac{1}{29400}\left[600 \cdot 10 \cdot \left(600 - \frac{10 + 20}{2}\right) + 2 \cdot 570 \cdot 10 \cdot \left(600 - \frac{570 + 20}{2}\right)\right]$$

$$S_y = 6.873 \cdot 10^6\ mm^3$$

$$r_t = \frac{S_y}{A} = \frac{6.873 \cdot 10^6}{29400.0} = 233.8\ mm$$

where:

S_y is the first moment of area of the gross cross section with respect to the centroid of the lower flange (y-y axis),

r_t is the distance from the centroid of the lower flange to the centroid of the gross cross-section

Calculation of effective^p width of the upper flange

$$\psi = 1.0 \rightarrow k_\sigma = 4.0$$

$$\bar{\lambda}_p = \frac{b_{f1}}{t_{f1} \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = \frac{580}{10 \cdot 28.4 \cdot 0.924 \cdot \sqrt{4.0}} = 1.105$$

Plate elements without longitudinal stiffeners, EN 1993-1-5, 4.4

$$\bar{\lambda}_p = 1.105 > 0.5 + \sqrt{0.085 - 0.055 \cdot \Psi}$$

$$= 0.5 + \sqrt{0.085 - 0.055 \cdot 1} = 0.673$$

$$\rho = \frac{\overline{\lambda}_p - 0.055 \cdot (3 + \Psi)}{\overline{\lambda}_p^2} = \frac{1.105 - 0.055 \cdot (3 + 1)}{1.105^2} = 0.725$$

$$b_{eff,f} = \rho \cdot b_{f1} = 0.725 \cdot 580 = 420.5\ mm$$

$$b_{e1,f} = b_{e2,f} = 0.5 \cdot b_{eff,f} = 0.5 \cdot 420.5 = 210.2\ mm$$

Calculation of effective^{p} width of the web

$$\psi = 1.0 \rightarrow k_\sigma = 4.0$$

Plate elements without longitudinal stiffeners, EN 1993-1-5, 4.4

$$\overline{\lambda}_p = \frac{h_w}{t_w \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = \frac{570}{10 \cdot 28.4 \cdot 0.924 \cdot \sqrt{4.0}} = 1.086$$

$$\overline{\lambda}_p = 1.086 > 0.5 + \sqrt{0.085 - 0.055 \cdot \varphi}$$

$$\overline{\lambda}_p = 1.086 > 0.5 + \sqrt{0.085 - 0.055 \cdot 1} = 0.673$$

$$\rho = \frac{\overline{\lambda}_p - 0.055 \cdot (3 + \varphi)}{\overline{\lambda}_p^2} = \frac{1.086 - 0.055 \cdot (3 + 1)}{1.086^2} = 0.734$$

$$b_{eff,w} = \rho \cdot h_w = 0.734 \cdot 570 = 418.7\ mm$$

$$b_{e1,w} = b_{e2,w} = 0.5 \cdot b_{eff,w} = 0.5 \cdot 418.7 = 209.3\ mm$$

Determination of characteristics of effective cross section considering effective widths of the upper flange and webs in uniform compression

$$x_f = b_{f1} - b_{eff,f} = 580.0 - 420.5 = 159.5\ mm$$

$$x_w = h_w - b_{eff,w} = 570.0 - 418.7 = 151.3\ mm$$

$$A_{eff} = \left[A - (x_f \cdot t_{f1} + 2 \cdot x_w \cdot t_w)\right]$$

$$A_{eff} = 29400 - (159.5 \cdot 10 + 2 \cdot 151.3 \cdot 10) = 24778.1\ mm^2$$

$$r_f = h - \frac{t_{f1} + t_{f2}}{2} - r_t = 600 - \frac{10 + 20}{2} = 351.2\ mm$$

$$r_w = h_w + \frac{t_{f2}}{2} - r_T - b_{e1,w} - \frac{x_w}{2}$$

$$r_w = 570 + \frac{10}{2} - 233.8 - 209.3 - \frac{151.3}{2} = 61.2\ mm$$

$$e_{N,y} = \frac{2 \cdot r_w \cdot x_w \cdot t_w + r_f \cdot x_f \cdot t_{f1}}{A_{eff}}$$

$$e_{N,y} = \frac{2 \cdot 61.2 \cdot 151.3 \cdot 10 + 351.2 \cdot 159.5 \cdot 10}{24778.1} = 30.1\ mm$$

$$r_{Teff,N} = r_T - e_{N,y} = 233.8 - 30.1 = 203.7\ mm$$

where:

$e_{N,y}$ is the shift of centroid of the effective area relative to the centre of gravity of the gross cross section determined assuming uniform axial compression.

$r_{Teff,N}$ is the distance from the centroid of the bottom flange to the centroid of the effective cross-section under uniform compression.

STEP 3: Calculating the effective properties assuming the cross-section is subject only to bending stresses

The effective section modulus $W_{eff,y}$ is determined on the cross-section subject only to bending moment.

Cross section class check

Upper flange (compression â€“ the same as in 1): Class 4

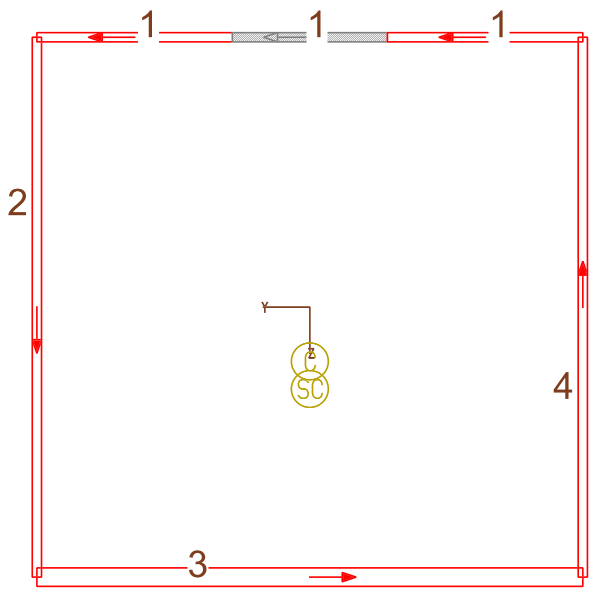

Figure 4: Effective area for bending

Determination of characteristics of effective cross section considering effective widths of the upper flange (calculation of effective width of the upper flange is already done in section 1) and gross cross section of the web:

$$A_{eff} = A - x_f \cdot t_{f1}$$

$$A_{eff} = 29400 - 159.5 \cdot 10 = 27804.9 \ mm$$

$$\Delta r_{T,m} = \frac{r_f \cdot x_f \cdot t_{f1}}{A_{eff}} = \frac{351.2 \cdot 159.5 \cdot 10}{24778.1} = 20.1 \ mm$$

$$r_{Teff,M} = r_T - \Delta r_{T,M} = 233.8 - 20.1 = 213.6 \ mm$$

$$I^I_{eff,y} = I_y + A_{eff} \cdot \Delta r^2_{T,M} - \left(\frac{x_f \cdot t_f^3}{12} + x_f \cdot t_f \cdot (r_f + \Delta r_{T,M})^2 \right)$$

$$I^I_{eff,y} = 1.748 \cdot 10^9 + 27804.9 \cdot 20.1^2 - \left(\frac{159.5 \cdot 10^3}{12} + \right.$$

$$159.5 \cdot 10 \cdot (351.2 + 20.1)^2\big) = 1.539 \cdot 10^9\ mm^4$$

where:

$I^I_{eff,y}$ is the effective second moment of area (cross section under pure bending) with respect to y-y considering the effective width of the upper flange.

The effective section moduli at the upper and lower edge of the girder's web, $W^I_{eff,y,1}$ and $W^I_{eff,y,2}$ are, respectively:

$$W^I_{eff,y,1} = \frac{I^I_{eff,y}}{h_w + \frac{t_{f2}}{2} - r_{Teff,M}}$$

$$W^I_{eff,y,1} = \frac{1.540 \cdot 10^9}{570 - \frac{20}{3} - 213.6} = 4.20 \cdot 10^6\ mm^3$$

$$W^I_{eff,y,2} = \frac{I^I_{eff,y}}{r_{Teff,M} - \frac{t_{f2}}{2}}$$

$$W^I_{eff,y,2} = \frac{1.540 \cdot 10^9}{213.6 - \frac{20}{2}} = 7.558 \cdot 10^6\ mm^3$$

Web (bending):

$$\Psi^I = \frac{\sigma^I_2}{\sigma^I_1} = \frac{M_{y,Ed}/W^I_{eff,y,2}}{M_{y,Ed}/W^I_{eff,y,1}} = \frac{W^I_{eff,y,1}}{W^I_{eff,z,2}} = \frac{4.20 \cdot 10^6}{7.558 \cdot 10^6}$$

$$\Psi^I = -0.56 > -1$$

$$\frac{h_w}{t_w} = \frac{570}{10} = 57.0 > \frac{42 \cdot \varepsilon}{0.67 + 0.33 \cdot \Psi^I} = \frac{42 \cdot 0.924}{0.67 - 0.33 \cdot 0.56}$$

$$\frac{h_w}{t_w} = 57.0 > 79.8 \text{ (Class 3)}$$

The web is at least of Class 3.

In case of a slender web, the effective^p width should be determined on the basis of stress ration Ψ^I.

The effective section modulus $W_{eff,y}$ for the design resistance to uniform bending is defined as the smallest value of the effective section moduli at the centroid of the upper and lower flange, $W_{eff,y,I}$ and $W_{eff,y,2}$, respectively:

$$W_{eff,y,1} = \frac{I^I_{eff,y}}{h_w + \frac{t_{f1} + t_{f2}}{2} - r_{Teff,M}} = \frac{1.540 \cdot 10^9}{570 - \frac{10 + 20}{2} - 213.6}$$

$$W_{eff,y,1} = 4.144 \cdot 10^6\ mm^3$$

$$W_{eff,y,2} = \frac{I^{I}_{eff,y}}{z_{Teff}} = \frac{1.540 \cdot 10^9}{213.6} = 7.205 \cdot 10^6\ mm^3$$

Here, $W_{eff,y,1}$ governs.

STEP 4: Cross section resistance check

Additional bending moment $N_{Ed} \cdot e_{N,y}$ causes compression at the upper flange (+compression).

$$\frac{N_{Ed}}{A_{eff} \cdot f_y / \gamma_{M0}} + \frac{N_{Ed} \cdot e_{N,y}}{W_{eff,y,1} \cdot f_y / \gamma_{M0}}$$

Class 4-sections, EN 1993-1-1, 6.2.9.3, Eq. 6.44

$$= \frac{5.5 \cdot 10^6}{24778.1 \cdot 275/1.0} + \frac{5.5 \cdot 10^6 \cdot 30.1}{4.144 \cdot 10^6 \cdot 275/1.0}$$

$$= 0.807 + 0.145$$

$= 0.95 < 1.0$ Satisfactory

2. STABILITY CHECK

In this example Method 2 is applied. Since the member has a rectangular hollow cross-section, the member is not susceptible to torsional deformation, so flexural buckling constitutes the relevant instability mode and $\chi_{LT} = 1.00$

STEP 1: Characteristic resistance of the section

$$N_{Rk} = A \cdot f_y = 24778.1 \cdot 275 = 6813977.5\ N = 6813.97\ kN$$

$$M_{y,Rk} = W_{pl,y} \cdot f_y = 4.144 \cdot 10^6 \cdot 275 = 1139.6 \cdot 10^{-6}\ Nm$$

$$M_{y,Rk} = 1139.6\ kNm$$

N_{Rk} is calculated assuming that the cross-section is subject only to stresses due to uniform axial compression, EN 1993-1-5, 4.3(4).

$M_{y,Rk}$ is calculated assuming the cross section is subject only to bending stresses, EN 1993-1-5, 4.3(4)

STEP 2: Reduction coefficients due to flexural buckling, χ_y and χ_z

Plane xz (buckling about y)

$$L_{cr,y} = \beta \cdot L = 4.00\ m$$

$$\bar{\lambda}_y = \frac{L_{cr,y}}{i_y} \cdot \frac{\sqrt{\frac{A_{eff}}{A}}}{\lambda_1}$$

EN 1993-1-1, Eq. 6.51

$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{174700.00}{294.00}} = 24.38\ cm$$

$$\lambda_1 = 93.9 \cdot \varepsilon$$

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.9244$$

$$\bar{\lambda}_y = \frac{400}{24.38} \cdot \frac{\sqrt{\frac{247.78}{294.00}}}{93.9 \cdot 0.9244}$$

EN 1993-1-1, Eq. 6.51

$$\bar{\lambda}_y = 0.173$$

Buckling curves, EN 1993-1-1, Table 6.2

$\alpha = 0.34$ Curve b

$$\phi = 0.5 \cdot \left[1 + \alpha \cdot (\overline{\lambda}_y - 0.2) + \overline{\lambda}_y^2\right]$$

$$\phi = \cdot 0.5 \left[1 + 0.34 \cdot (0.173 - 0.2) + 0.173^2\right]$$

$$\phi = 0.51$$

χ, ϕ, EN 1993-1-1, Eq. 6.49

$$\chi_y = \frac{1}{\phi + \sqrt{\phi^2 - \overline{\lambda}_y^2}} \leq 1.0$$

$$\chi_y = \frac{1}{0.51 + \sqrt{0.51^2 - 0.173^2}}$$

$$\chi_y = 1.01 \leq 1.0 \rightarrow \chi_y = 1.0$$

Plane xy (buckling about z):

$$L_{cr,z} = \beta \cdot L = 4.00\ m$$

EN 1993-1-1, Eq. 6.51

$$\overline{\lambda}_z = \frac{L_{cr,z}}{i_z} \cdot \frac{\sqrt{\frac{A_{eff}}{A}}}{\lambda_1}$$

$$i_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{153200.00}{294.00}} = 22.82\ cm$$

EN 1993-1-1, Eq. 6.51

$$\overline{\lambda}_z = \frac{400}{22.82} \cdot \frac{\sqrt{\frac{247.78}{294.00}}}{93.9 \cdot 0.9244}$$

$$\overline{\lambda}_z = 0.185$$

Buckling curves, EN 1993-1-1, Table 6.2

$\alpha = 0.34$ Curve b

$$\phi = 0.5 \cdot \left[1 + \alpha \cdot (\overline{\lambda}_z - 0.2) + \overline{\lambda}_z^2\right]$$

$$\phi = \cdot 0.5 \left[1 + 0.34 \cdot (0.185 - 0.2) + 0.185^2\right]$$

$$\phi = 0.51$$

χ, ϕ, EN 1993-1-1, Eq. 6.49

$$\chi_z = \frac{1}{\phi + \sqrt{\phi^2 - \overline{\lambda}_z^2}} \leq 1.0$$

$$\chi_z = \frac{1}{0.51 + \sqrt{0.51^2 - 0.173^2}}$$

$$\chi_z = 1.01 \leq 1.0 \rightarrow \chi_z = 1.0$$

STEP 3: Calculating of the interaction factors k_{yy} and k_{zy}

$$\psi_y = M_{y,Ed,base} / M_{Ed,top} = 175.5/175.5 = 1.0$$

Table B.3 of EN 1993-1-1 gives:

Interaction factors $k_{i,j}$ for members not susceptible to torsional deformations, Annex B, Table B.1

$$C_{my} = 0.6 + 0.4 \cdot \psi_y \geq 0.4$$

$C_{my} = 0.6 + 0.4 \cdot 1.0 = 1.0$

$$k_{yy} = C_{my} \cdot \left(1 + 0.6 \cdot \bar{\lambda}_y \cdot \frac{N_{Ed}}{\chi_y \cdot N_{Rk}}\right) \leq C_{my} \cdot \left(1 + 0.6 \cdot \frac{N_{Ed}}{\chi_y \cdot N_{Rk}}\right)$$

$$k_{yy} = 1.0 \cdot \left(1 + 0.6 \cdot 0.173 \cdot \frac{5500}{1.0 \cdot 6813.97}\right)$$
$$\leq 1.0 \cdot \left(1 + 0.6 \cdot \frac{5500}{0.173 \cdot 6813.97}\right)$$

$k_{yy} = 1.0837 \leq 1.484$

$k_{zy} = 0.8 \cdot k_{yy} = 1.084 \cdot 0.8 = 0.867$

Final expression

Check for y-y

Uniform members in bending and axial compression, EN 1993-1-1, 6.3.3, Eq. 6.61

$$\frac{N_{Ed}}{\chi_y \cdot \frac{N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{y,Ed}}{\chi_{LT} \cdot \frac{M_{y,Rk}}{\gamma_{M1}}} \leq 1.0$$

$$\frac{5500}{1.0 \cdot \frac{6813.9}{1.0}} + 1.084 \cdot \frac{175}{1.0 \cdot \frac{1139.6}{1.0}} \leq 1.0$$

$0.807 + 0.166 = 0.973 \leq 1.0 \rightarrow$ Satisfied

Check for z-z

Uniform members in bending and axial compression, EN 1993-1-1, 6.3.3, Eq. 6.62

$$\frac{N_{Ed}}{\chi_z \cdot \frac{N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \frac{M_{y,Ed}}{\chi_{LT} \cdot \frac{M_{y,Rk}}{\gamma_{M1}}} \leq 1.0$$

$$\frac{5500}{1.0 \cdot \frac{6813.9}{1.0}} + 0.867 \cdot \frac{175}{1.0 \cdot \frac{1139.6}{1.0}} \leq 1.0$$

$0.807 + 0.133 = 0.94 \leq 1.0 \rightarrow$ Satisfied

5 Conclusion

In the reference example, the effective area A_{eff} is determined assuming that the cross-section is subjected only to stresses due to uniform axial compression (EN 1993-1-5, 4.3(3)) $A_{c,eff} = \rho \cdot A_c$. The effective section modulus W_{eff} is determined assuming the cross-section is subject to only bending stresses (EN 1993-1-5, 4.3(3)).

By using the NEFF SIG SMIN input it is possible to define only one effective cross-section for the design and stability check, therefore the effective section modulus is determined assuming that the cross-section is subject only to stresses due uniform axial compression. The A_{eff} as well as $W_{eff,y}$ and $W_{eff,z}$ values are calculated in SOFiSTiK for the effective cross-section as shown in Fig. 3. This approach checks the MOST UNFAVOURABLE case where all plates are under compression.

By using the iterative method (EN 1993-1-5, Annex E) for calculating the effective cross-section properties, the effective CS properties will be calculated for the current stress state, so it gives more realistic and economical results as shown in table 2. The iterative method can be used ONLY for the THIN-WALLED cross-sections. In Fig. 5 you will find the comparison between "SIG NEFF", "Iterative approach" and the reference.

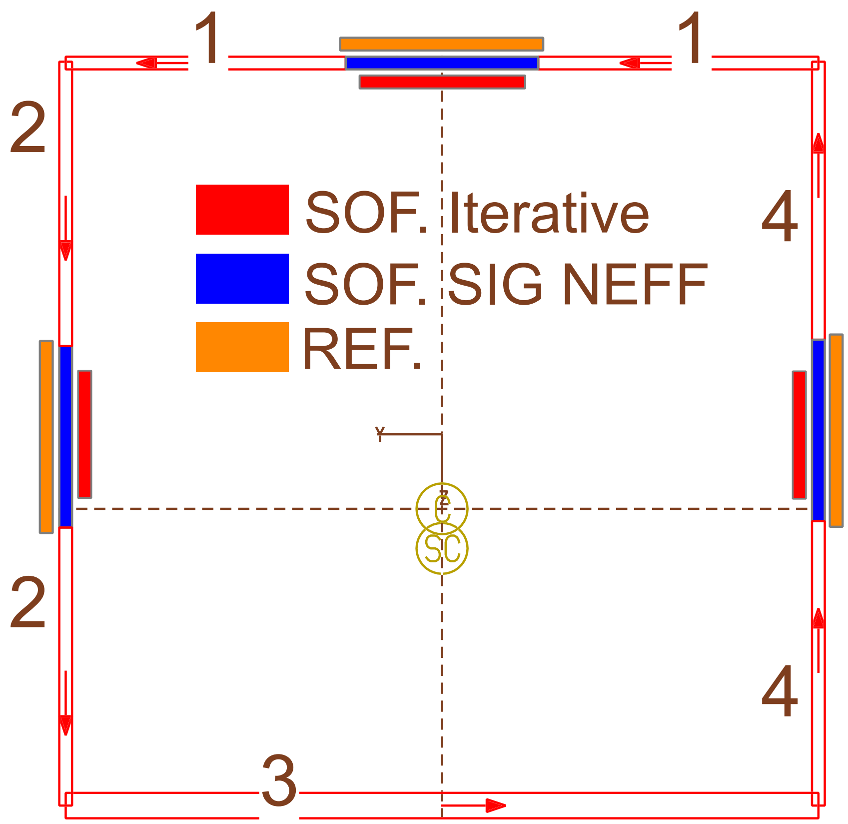

Figure 5: Comparison of the $b_{i,eff}$ values

6 Literature

[1] D. Beg et al. *Design of Plated Structures*. Ernst & Sohn and ECCS, 2010.

[2] *EN 1993-1-1: Eurocode 3: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2005.

[3] *EN 1993-1-5: Eurocode 3: Design of steel structures, Part 1-5: Plated structural elements*. CEN. 2006.

[4] *DIN EN 1993-1-1:2005 Eurocode 3: Design of steel structures, Part 1-1: General rules and rules for buildings - Deutsche Fassung EN 1993-1-1:2005 + AC:2009*. CEN. 2010.