Benchmark Example No. 27

Design of Quad Elements - Layer Design and Baumann Method

SOFiSTiK | 2022

VERiFiCATiON
DCE-EN27 Design of Quad Elements - Layer Design and Baumann Method

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Front Cover

Arnulfsteg, Munich Photo: Hans Gössing

Overview	
Design Code Family(s):	EN
Design Code(s):	EN 1992-1-1
Module(s):	BEMESS
Input file(s):	layer_design_baumann.dat

1 Problem Description

The problem consists of a one-way slab, as shown in Fig. 1 . The slab is designed for bending. This benchmark presents a procedure which uses the model based on Baumann's criteria and the Layer Design approach.

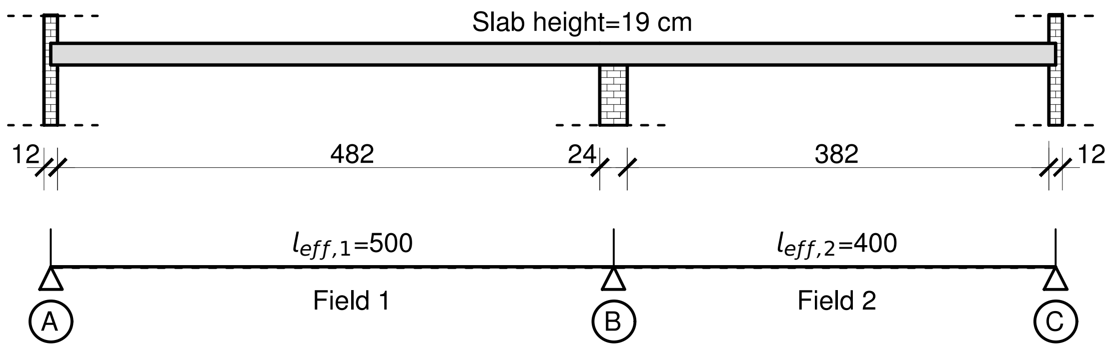

Figure 1: Problem Description (in cm)

2 Reference Solution

This example is concerned with the design of a one-way slab, for the ultimate limit state. The content of this problem is covered by the following parts of EN 1992-1-1:2004 [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Bending with or without axial force (Section 6.1)

The verification of the BEMESS results will be examined. A complete and detailed hand-calculation of the results is not possible because of described BEMESS-strategy, which should be here to exhaustive. For this reason, some results (e.g. internal forces) will be taken as outputted and further used in the hand-calculation.

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 2 and as defined in EN 1992-1-1:2004 [2] (Section 3.2.7).

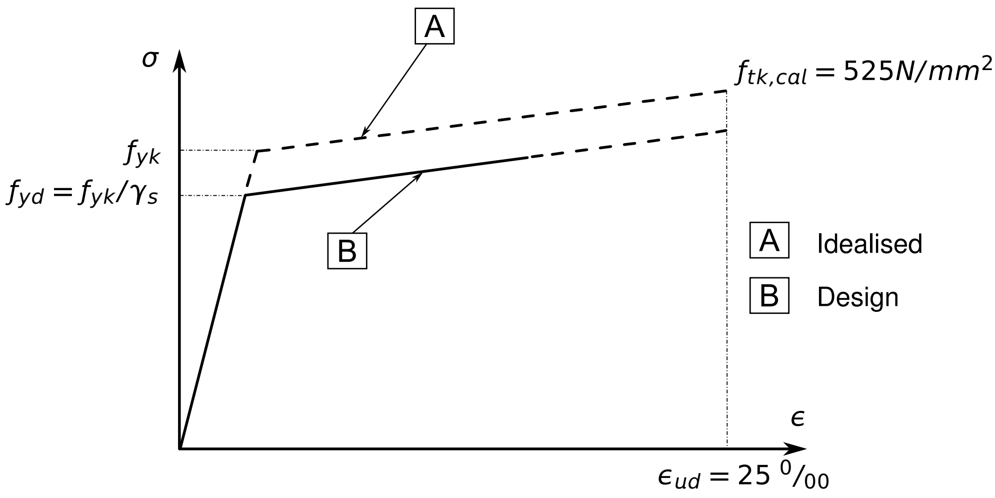

Figure 2: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The slab, with properties as defined in Table 3, is to be designed for bending moment, with respect to EN 1992-1-1:2004 [2]. The structure analysed, consists of one-way slab with a distributed load in gravity direction. The loading is presented bellow:

Table 1: Loading and actions

Title	Action	Safety Factor ULS	Value
Dead Load	G dead load	$\gamma_G = 1.35$	$g_k = 6.35\ kN/m^2$
Field 1	Q variable load	$\gamma_Q = 1.5$	$q_k = 5.0\ kN/m^2$
Field 2	Q variable load	$\gamma_Q = 1.5$	$q_k = 5.0\ kN/m^2$

$g_d = \gamma_G \cdot g_k = 1.35 \cdot 6.35 = 8.57\ kN/m^2$

$q_{d,f1} = \gamma_Q \cdot q_k = 1.50 \cdot 5.00 = 7.50\ kN/m^2$

$q_{d,f2} = \gamma_Q \cdot q_k = 1.50 \cdot 5.00 = 7.50\ kN/m^2$

LC 1001 $\rightarrow g_d + q_d$ in Field 1+2

LC 1002 $\rightarrow g_d + q_{d,f1}$ in Field 1

LC 1003 $\rightarrow g_d + q_{d,f2}$ in Field 2

Table 2: Internal forces

Loadcase	$m_{Ed,B}$	$m_{Ed,F1}$	$m_{Ed,F2}$	$v_{Ed,A}$	$v_{Ed,B,l}$	$v_{Ed,B,r}$	$v_{Ed,C}$
LC 1001	-37.16	31.00	14.40	30.00	-46.80	41.00	-20.00
LC 1002	-35.20	33.60	4.03	31.30	-45.40	25.10	-7.49
LC 1003	-28.90	14.10	19.09	14.70	-26.30	37.70	-23.20

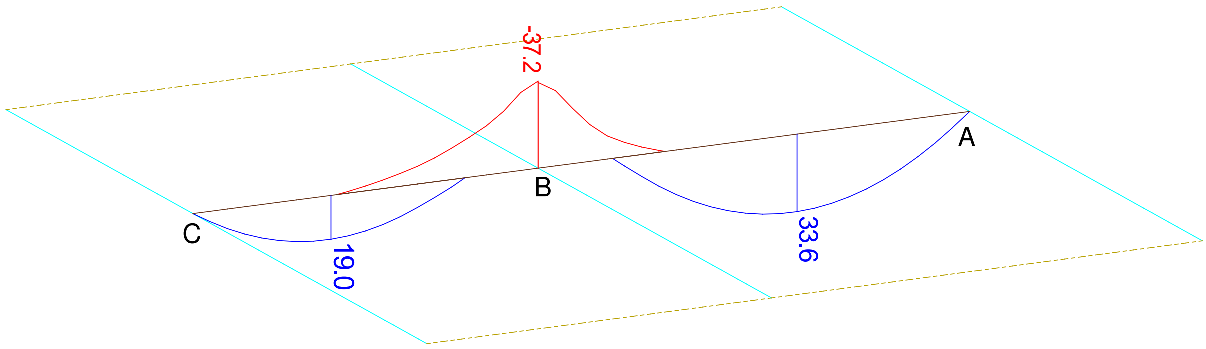

Figure 3: m_{Ed} envelope in kNm/m^2

Table 3: Model Properties

Material Properties	Geometric Properties	Loading
$C\ 20/25$	$h = 19\ cm$	$g_d = 8.57\ kN/m^2$
$B\ 500B,\ B\ 500A$	$c_{nom} = 20\ mm$	$q_{d,f1} = 7.50\ kN/m^2$
	$d_1 = 3.0\ cm$	$q_{d,f2} = 7.50\ kN/m^2$
	Exposition class XC1	

The system with its loading are shown in Fig. 4-9. The reference calculation steps are presented in the next section and the results are given in Table 4.

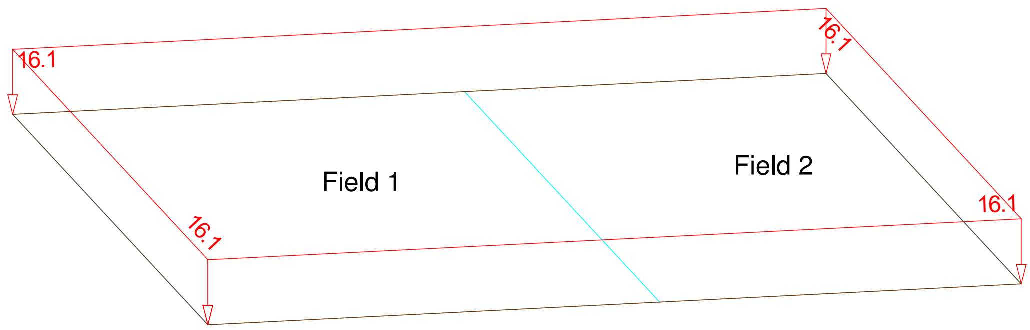

Figure 4: Loadcase 1001

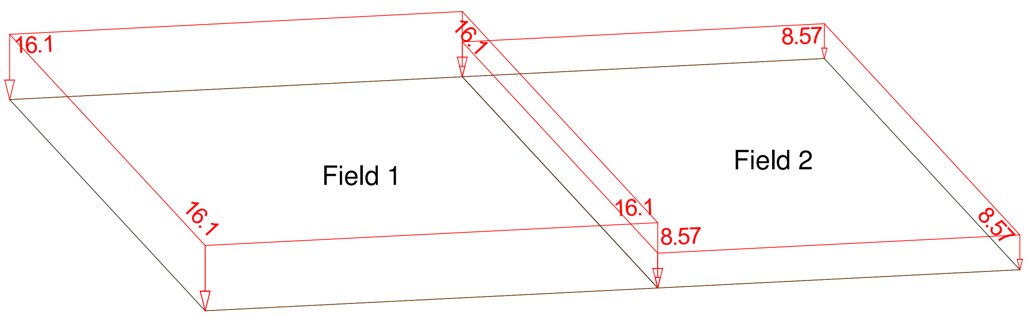

Figure 5: Loadcase 1002

QUAD 10026 over Support B:

Required Reinforcements acc. to EN 1992-1-1:2004

Grp	Element	LC	t [m]	asu [cm2/m]	asu2 [cm2/m]	asu3 [cm2/m]	asl [cm2/m]	asl2 [cm2/m]	asl3 [cm2/m]	supp [-]	shear [-]	ass [cm2/m2]
1	10026	1001	0.190	5.57	1.14					0.00	1	

```
Grp      primary group number                         asu2  Cross reinforcements (2nd layer)      Top
Element  element number                               asu3  Third reinforcements                  Top
LC       load case                                    asl   Principal reinforcements (1st layer) Bottom
t        plate thickness                              asl2  Cross reinforcements (2nd layer)      Bottom
asu      Principal reinforcements (1st layer) Top     asl3  Third reinforcements                  Bottom
supp     reduction factor for the shear force near supports, punc=point in punching zone -> punching shear design
shear    shear zone: 1=Ok, punc=punching area, 1s=asu/l increased for shear, 1d=for punching, 2=required ass, 2m=minimum shear reinf.
ass      Shear reinforcement
```

Figure 6: Layer Design - Quad 10026, Loadcase 1001, $a_{st} = 5.57\ cm^2/m$

QUAD 10726 in Field 1:

Required Reinforcements acc. to EN 1992-1-1:2004

Grp	Element	LC	t [m]	asu [cm2/m]	asu2 [cm2/m]	asu3 [cm2/m]	asl [cm2/m]	asl2 [cm2/m]	asl3 [cm2/m]	supp [-]	shear [-]	ass [cm2/m2]
1	10726	1002	0.190				4.94	0.97			1	

```
Grp      primary group number                         asu2  Cross reinforcements (2nd layer)      Top
Element  element number                               asu3  Third reinforcements                  Top
LC       load case                                    asl   Principal reinforcements (1st layer) Bottom
t        plate thickness                              asl2  Cross reinforcements (2nd layer)      Bottom
asu      Principal reinforcements (1st layer) Top     asl3  Third reinforcements                  Bottom
supp     reduction factor for the shear force near supports, punc=point in punching zone -> punching shear design
shear    shear zone: 1=Ok, punc=punching area, 1s=asu/l increased for shear, 1d=for punching, 2=required ass, 2m=minimum shear reinf.
ass      Shear reinforcement
```

Figure 7: Layer Design - Quad 10726, Loadcase 1002, $a_{st} = 4.94\ cm^2/m$

QUAD 80376 in Field 2:

Required Reinforcements acc. to EN 1992-1-1:2004

Grp	Element	LC	t [m]	asu [cm2/m]	asu2 [cm2/m]	asu3 [cm2/m]	asl [cm2/m]	asl2 [cm2/m]	asl3 [cm2/m]	supp [-]	shear [-]	ass [cm2/m2]
8	80376	1003	0.190				2.76	0.57			1	

```
Grp      primary group number                         asu2  Cross reinforcements (2nd layer)      Top
Element  element number                               asu3  Third reinforcements                  Top
LC       load case                                    asl   Principal reinforcements (1st layer) Bottom
t        plate thickness                              asl2  Cross reinforcements (2nd layer)      Bottom
asu      Principal reinforcements (1st layer) Top     asl3  Third reinforcements                  Bottom
supp     reduction factor for the shear force near supports, punc=point in punching zone -> punching shear design
shear    shear zone: 1=Ok, punc=punching area, 1s=asu/l increased for shear, 1d=for punching, 2=required ass, 2m=minimum shear reinf.
ass      Shear reinforcement
```

Figure 8: Layer Design - Quad 80376, Loadcase 1003, $a_{st} = 2.76\ cm^2/m$

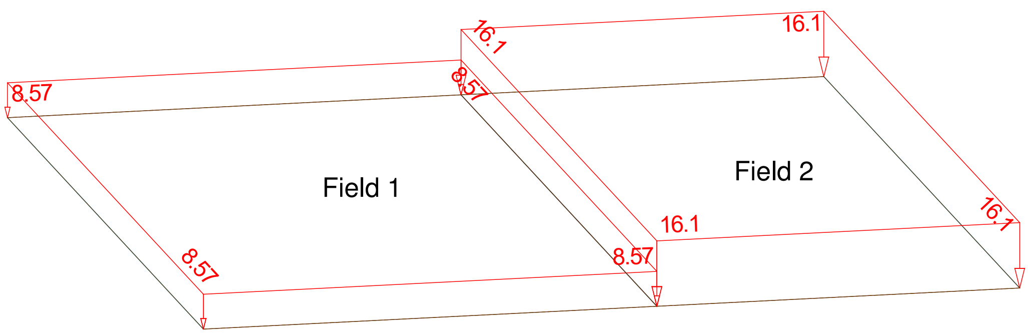

Figure 9: Loadcase 1003

Table 4: Results $a_s [cm^2/m]$

Quad	SOFiSTiK		Reference	
	Baumann	Layer Design	Baumann	Tables
Support B (QUAD 10026)	5.52	5.58	5.38	5.45
Field 1 (QUAD 10726)	4.95	4.97	4.88	4.88
Field 2 (QUAD 80376)	2.65	2.81	2.70	2.70

4 Design Process[1]

Design with respect to EN 1992-1-1:2004 [2] :[2]

Material:

2.4.2.4: (1), Tab. 2.1N: Partial factors for materials for ultimate limit states

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

Tab. 3.1: Strength and deformation characteristics for concrete

3.1.6: Eq. (3.15): $\alpha_{cc} = 1.00$ considering long term effects

3.2.2: (3)P: yield strength $f_{yk} = 500$ MPa

3.2.7: (2), Fig. 3.8

$f_{ck} = 20\ MPa$
$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 1.00 \cdot 20/1.50 = 13.33 MPa$

$f_{yk} = 500\ MPa$
$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

$\sigma_{sd} = 456.52\ MPa$

1. DESIGN BY USING TABLES

To make the example more simple, the slab will be designed only for the maximum and minimum moment m_{Ed} (Field 1, Field 2 and over the middle support). The reduction of the moment over the middle support will be neglected in this example.

• Design m_{Ed} over the middle support (QUAD 10026):

$$m_{Ed,\mathrm{B}} = (m_{Ed,B,1001}, m_{Ed,B,1002}, m_{Ed,B,1003})$$

$$m_{Ed,\mathrm{B}} = \max(-37.16,\ -35.20,\ -28.90) = -37.16\ kNm/m$$

Calculating the μ value:

$$\mu_{Eds} = \frac{|m_{Ed,B,Red}|}{b \cdot d^2 \cdot f_{cd}}$$

$$\mu_{Eds} = \frac{37.16 \cdot 10^{-3}}{1.0 \cdot 0.16^2 \cdot 13.33}$$

$$\mu_{Eds} = 0.1089$$

From tables:

μ	ω	ξ	ζ	σ_{sd} [MPa]
0.108867	0.11575	0.14298	0.940522	452.69

$$a_{s,req} = \omega \cdot b \cdot d \cdot f_{cd} \cdot \frac{1}{\sigma_{sd}}$$

$$a_{s,req} = 0.11576 \cdot 1.00 \cdot 0.16 \cdot 13.33 \cdot \frac{1}{452.70}$$

$$a_{s,req} = 5.45\ cm^2/m$$

• Design m_{Ed} over in Field 1 (QUAD 10726):

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [3] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 2.

[2] The sections mentioned in the margins refer to EN 1992-1-1:2004 [2], unless otherwise specified.

$m_{Ed,f1} = (m_{Ed,f1,1001}, m_{Ed,f1,1002}, m_{Ed,f1,1003})$

$m_{Ed,f1} = \max(31.00,\ 33.69,\ 14.10) = 33.69\ kNm/m$

Calculating the μ value:

$$\mu_{Eds} = \frac{|m_{Ed,B,Red}|}{b \cdot d^2 \cdot f_{cd}}$$

$$\mu_{Eds} = \frac{33.69 \cdot 10^{-3}}{1.0 \cdot 0.16^2 \cdot 13.33}$$

$$\mu_{Eds} = 0.0987$$

From tables:

μ	ω	ξ	ζ	σ_{sd} [MPa]
0.0987	0.10429	0.12882	0.9464	455.25

$$a_{s,req} = \omega \cdot b \cdot d \cdot f_{cd} \cdot \frac{1}{\sigma_{sd}}$$

$$a_{s,req} = 0.10429 \cdot 1.00 \cdot 0.16 \cdot 13.33 \cdot \frac{1}{455.25}$$

$$a_{s,req} = 4.88\ cm^2/m$$

- Design m_{Ed} over in Field 2 (QUAD 80376):

$m_{Ed,f2} = (m_{Ed,f2,1001}, m_{Ed,f2,1002}, m_{Ed,f2,1003})$

$m_{Ed,f2} = \max(14.40,\ 4.03,\ 19.09) = 19.09\ kNm/m$

Calculating the μ value:

$$\mu_{Eds} = \frac{|m_{Ed,B,Red}|}{b \cdot d^2 \cdot f_{cd}}$$

$$\mu_{Eds} = \frac{19.09 \cdot 10^{-3}}{1.0 \cdot 0.16^2 \cdot 13.33}$$

$$\mu_{Eds} = 0.05593$$

From tables:

μ	ω	ξ	ζ	σ_{sd} [MPa]
0.05593	0.05774	0.08222	0.968533	456.52

$$a_{s,req} = \omega \cdot b \cdot d \cdot f_{cd} \cdot \frac{1}{\sigma_{sd}}$$

$$a_{s,req} = 0.05774 \cdot 1.00 \cdot 0.16 \cdot 13.33 \cdot \frac{1}{456.52}$$

$$a_{s,req} = 2.698\ cm^2/m$$

2. DESIGN BY USING THE MULTI LAYER APPROACH

The design approach in BEMESS 2018 was completely changed from

Baumann Method to an exact iteration of the strain state. This iteration of the stain state is called ”Layer Design” or ”Layer Approach” in SOFiSTIK. In the layer design the 6 strain parameters (3 strains ε_x, ε_y, ε_{xy}) and 3 curvatures k_x, k_y, k_{xy}) are calculated iteratively to achieve equilibrium between the 6 inner forces and the 6 internal forces n_x, n_y, n_{xy}, m_x, m_y and m_{xy}. Thereby non-linear work-laws are taken into account for concrete and steel.

The iterative approach is not documented in this example, because it takes lot of effort to document all iterational steps. Therefore only the output is shown, see the output tables in Fig. 6, 7 and 8.

3. DESIGN BY USING BAUMANN METHOD

For each reinforcement layer: With the use of internal forces in local element direction, the internal forces in main direction and the accompaying angle is calculted with the following equation

$$m_{I/II} = \frac{m_{xx} + m_{yy}}{2} \pm 0.5 \cdot \sqrt{(m_{yy} - m_{xx})^2 + 4 \cdot m_{xy}^2}$$

$$\tan 2\varphi_0 = 2 \cdot \frac{m_{xy}}{m_{xx} - m_{yy}}$$

Accompanying to the main bending moments the normal forces and accompanying to the main normal forces the bending moments are calculated by transformation.

The internal lever arm is calculated separately for the internal forces in main moment direction and for the internal forces in main normal forces direction. This is done with the theory explained in the paper from Prof. Dr. Ing. Ulrich P. Schmitz [4]. The program choose the unfavorable lever arm from both results for the next analysis step lever arm z (This is why for each layer of reinforcement two lever arms are calculated within the program).

This lever arm z is used to calculate the virtual panel forces N_x, N_y, N_{xy} (in direction of the local element coordinate system) on each side of the finite element:

QUAD 10026 over Support B:

Calculating N_x:

$$N_x = \frac{n_{xx}}{2} + \frac{m_{xx}}{z}$$

$$N_x = \frac{0.00}{2} + \frac{37.16}{0.1514}$$

$$N_x = 245.442\ kN/m$$

Calculating N_y:

$$N_y = \frac{n_{yy}}{2} + \frac{m_{yy}}{z}$$

$$N_y = \frac{0.00}{2} + \frac{7.43}{0.1514}$$

$N_y = 49.075\ kN/m$

Calculating N_{xy}:

$$N_{xy} = \frac{n_{xy}}{2} + \frac{n_{xy}}{z}$$

$$N_{xy} = \frac{0.00}{2} + \frac{0.00}{0.1514}$$

$$N_{xy} = 0.00\ kN/m$$

The next step is the transformation of the panel forces N_x, N_y, N_{xy} into the main forces N_I and N_{II}:

$$N_I = \frac{N_x + N_y}{2} + 0.5 \cdot \sqrt{(N_y - N_x)^2 + 4 \cdot N_{xy}^2}$$

$$N_I = \frac{245.44 + 49.07}{2} + 0.5 \cdot \sqrt{(49.07 - 245.4)^2 + 4 \cdot 0.00^2}$$

$$N_I = 147.256 + 98.165 = 245.421$$

$$N_{II} = \frac{N_x + N_y}{2} - 0.5 \cdot \sqrt{(N_y - N_x)^2 + 4 \cdot N_{xy}^2}$$

$$N_{II} = \frac{245.44 + 49.07}{2} - 0.5 \cdot \sqrt{(49.07 - 245.4)^2 + 4 \cdot 0.00^2}$$

$$N_{II} = 147.256 - 98.165 = 49.091$$

$$\tan 2\varphi_0 = 2 \cdot \frac{N_{xy}}{N_x - N_y}$$

$$\tan 2\varphi_0 = 2 \cdot \frac{0.00}{245.442 - 49.07}$$

$$\tan 2\varphi_0 = 0.00 \rightarrow \varphi = 0$$

Required reinforcement:

$$k = \frac{N_2}{N_1}$$

$$k = \frac{49.091}{245.41} = 0.20$$

$$k \geq \tan(\alpha + \pi/4) \cdot \tan\alpha = 0$$

$$Z_x = N_I + \frac{N_I - N_{II}}{2} \cdot \sin 2\alpha \cdot (1 - \tan\alpha)$$

$$Z_x = 245.421 + \frac{245.421 - 49.091}{2} \cdot \sin 0 \cdot (1 - \tan 0)$$

$$Z_x = 245.421$$

$$a_s = \frac{Z_x}{\sigma_{sd}}$$

$$a_s = \frac{245.421}{456.52} = 5.375\ cm^2/m$$

Field 1 (QUAD 10726):

Calculating N_x:

$$N_x = \frac{n_{xx}}{z} + \frac{m_{xx}}{z}$$

$$N_x = \frac{0.00}{0.1514} + \frac{33.70}{0.1514}$$

$$N_x = 222.589\ kN/m$$

Calculating N_y:

$$N_y = \frac{n_{yy}}{z} + \frac{m_{yy}}{z}$$

$$N_y = \frac{0.00}{0.1514} + \frac{6.74}{0.1514}$$

$$N_y = 44.51\ kN/m$$

Calculating N_{xy}:

$$N_{xy} = \frac{n_{xy}}{z} + \frac{n_{xy}}{z}$$

$$N_{xy} = \frac{0.00}{0.1514} + \frac{0.00}{0.1514}$$

$$N_{xy} = 0.00\ kN/m$$

The next step is the transformation of the panel forces N_x, N_y, N_{xy} into the main forces N_I and N_{II}:

$$N_I = \frac{N_x + N_y}{2} + 0.5 \cdot \sqrt{(N_y - N_x)^2 + 4 \cdot N_{xy}^2}$$

$$N_I = \frac{222.589 + 44.51}{2} + 0.5 \cdot \sqrt{(44.51 - 222.589)^2 + 4 \cdot 0.00^2}$$

$$N_I = 133.5495 + 89.0395 = 222.589$$

$$N_{II} = \frac{N_x + N_y}{2} - 0.5 \cdot \sqrt{(N_y - N_x)^2 + 4 \cdot N_{xy}^2}$$

$$N_{II} = \frac{222.589 + 44.51}{2} - 0.5 \cdot \sqrt{(44.51 - 222.589)^2 + 4 \cdot 0.00^2}$$

$$N_{II} = 133.5495 - 89.0395 = 44.51$$

$$\tan 2\varphi_0 = 2 \cdot \frac{N_{xy}}{N_x - N_y}$$

$$\tan 2\varphi_0 = 2 \cdot \frac{0.00}{222.589 - 44.51}$$

$$\tan 2\varphi_0 = 0.00 \rightarrow \varphi = 0$$

Required reinforcement:

$$k = \frac{N_2}{N_1}$$

$$k = \frac{44.51}{222.589} = 0.199$$

$$k \geq \tan(\alpha + \pi/4) \cdot \tan\alpha = 0$$

$$Z_x = N_I + \frac{N_I - N_{II}}{2} \cdot \sin 2\alpha \cdot (1 - \tan\alpha)$$

$$Z_x = 222.589 + \frac{222.589 - 44.51}{2} \cdot \sin 0 \cdot (1 - \tan 0)$$

$$Z_x = 222.589$$

$$a_s = \frac{Z_x}{\sigma_{sd}}$$

$$a_s = \frac{222.589}{456.52} = 4.875\ cm^2/m$$

Field 2 (QUAD 80376):

Calculating N_x:

$$N_x = \frac{n_{xx}}{z} + \frac{m_{xx}}{z}$$

$$N_x = \frac{0.00}{0.1514} + \frac{19.09}{0.1549}$$

$$N_x = 123.24\ kN/m$$

Calculating N_y:

$$N_y = \frac{n_{yy}}{z} + \frac{m_{yy}}{z}$$

$$N_y = \frac{0.00}{0.1549} + \frac{3.86}{0.1549}$$

$$N_y = 24.91\ kN/m$$

Calculating N_{xy}:

$$N_{xy} = \frac{n_{xy}}{z} + \frac{n_{xy}}{z}$$

$$N_{xy} = \frac{0.00}{0.1549} + \frac{0.00}{0.1549}$$

$$N_{xy} = 0.00\ kN/m$$

The next step is the transformation of the panel forces N_x, N_y, N_{xy} into the main forces N_I and N_{II}:

$$N_I = \frac{N_x + N_y}{2} + 0.5 \cdot \sqrt{(N_y - N_x)^2 + 4 \cdot N_{xy}^2}$$

$$N_I = \frac{123.24 + 24.91}{2} + 0.5 \cdot \sqrt{(24.91 - 123.24)^2 + 4 \cdot 0.00^2}$$

$$N_I = 74.075 + 49.165 = 123.24$$

$$N_{II} = \frac{N_x + N_y}{2} - 0.5 \cdot \sqrt{(N_y - N_x)^2 + 4 \cdot N_{xy}^2}$$

$$N_{II} = \frac{123.24 + 24.91}{2} - 0.5 \cdot \sqrt{(24.91 - 123.24)^2 + 4 \cdot 0.00^2}$$

$$N_{II} = 74.075 - 49.165 = 24.91$$

$$\tan 2\varphi_0 = 2 \cdot \frac{N_{xy}}{N_x - N_y}$$

$$\tan 2\varphi_0 = 2 \cdot \frac{0.00}{123.24 - 24.91}$$

$$\tan 2\varphi_0 = 0.00 \rightarrow \varphi = 0$$

Required reinforcement:

$$k = \frac{N_2}{N_1}$$

$$k = \frac{24.91}{123.24} = 0.202$$

$$k \geq \tan(\alpha + \pi/4) \cdot \tan\alpha = 0$$

$$Z_x = N_I + \frac{N_I - N_{II}}{2} \cdot \sin 2\alpha \cdot (1 - \tan\alpha)$$

$$Z_x = 123.24 + \frac{123.24 - 24.91}{2} \cdot \sin 0 \cdot (1 - \tan 0)$$

$$Z_x = 123.24$$

$$a_s = \frac{Z_x}{\sigma_{sd}}$$

$$a_s = \frac{123.24}{456.52} = 2.70\ cm^2/m$$

5 Conclusion

This example shows the calculation of the required reinforcement for a one-way slab under bending. It has been shown that the results are reproduced with very good accuracy.

6 Literature

[1] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.

[2] *EN 1992-1-1: Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2004.

[3] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[4] Ulrich P. Schmitz. “Biegebemessung nach der neuen E-DIN 1045-1”. In: *Bauinformatik JOURNAL* (1999), pp. 49–51.