Benchmark Example No. 24

Lateral Torsional Buckling

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VERiFiCATiON
DCE-EN24 Lateral Torsional Buckling

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Front Cover

Arnulfsteg, Munich Photo: Hans Gössing

Overview	
Design Code Family(s):	EN
Design Code(s):	EN 1993-1-1
Module(s):	BDK
Input file(s):	eccs_119_example_5.dat

1 Problem Description

The problem consists of a simply supported beam with a steel I-section, which is subjected to compression and biaxial bending, as shown in Fig. 1. The beam is checked against lateral torsional buckling.

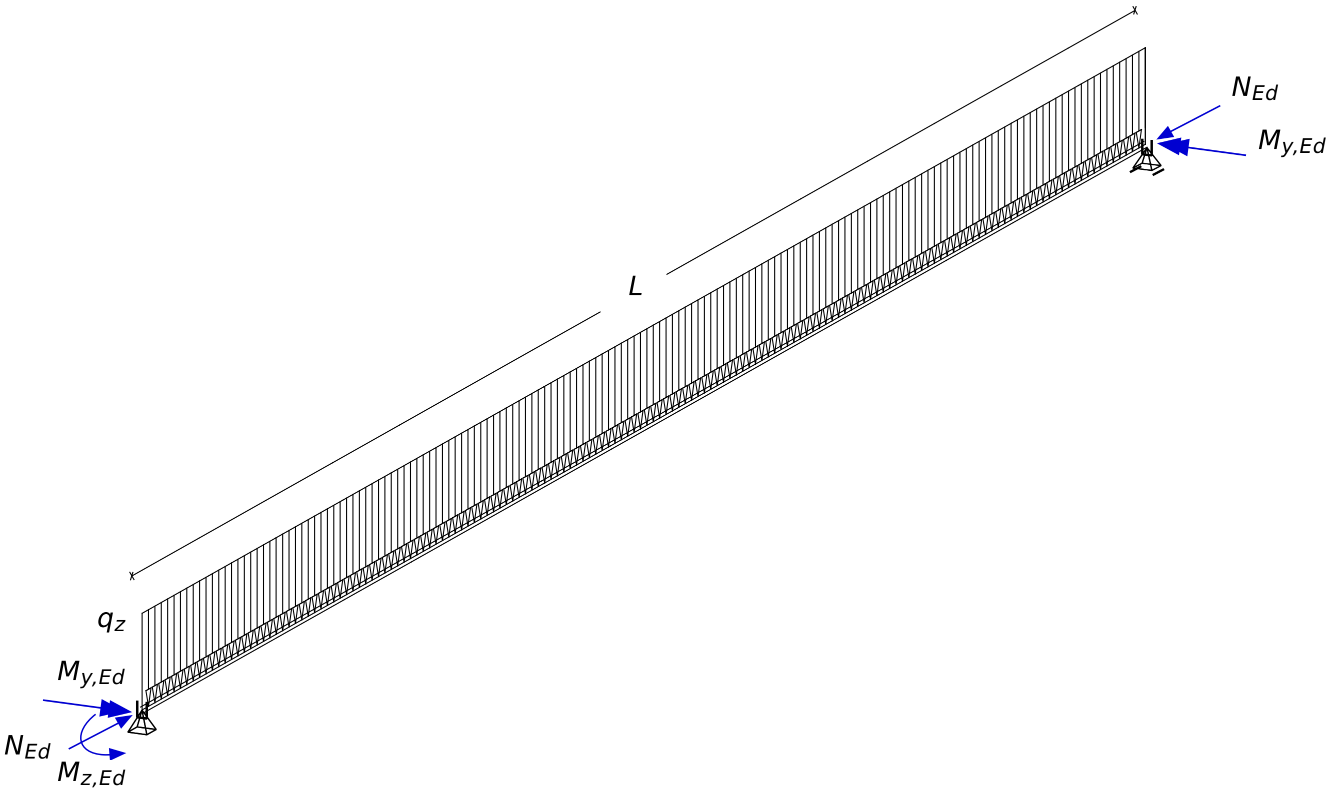

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the buckling resistance of steel members. It deals with the spatial behavior of the beam and the occurrence of lateral torsional buckling as a potential mode of failure. The content of this problem is covered by the following parts of EN 1993-1-1:2005 [1]:

- Structural steel (Section 3.2)
- Classification of cross-sections (Section 5.5)
- Buckling resistance of members (Section 6.3)
- Method 1: Interaction factors k_{ij} for interaction formula in 6.3.3(4) (Annex A)

3 Model and Results

The I-section, an IPE 500, with properties as defined in Table 1, is to be checked for lateral torsional buckling, with respect to EN 1993-1-1:2005 [1]. The calculation steps are presented below. The results are tabulated in Table 2 and compared to the results of reference [2].

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$S\ 235$	IPE 500	$M_{y,Ed} = 100\ kNm$
$E = 210000\ N/mm^2$	$L = 3.750\ m$	$M_{z,Ed} = 25\ kNm$
$\gamma_{M1} = 1.0$	$h_w = 468\ mm$	$q_z = 170\ kN/m$
	$b_f = 200\ mm$	$N_{Ed} = 500\ kN$
	$t_f = 16.0\ mm$	
	$t_w = 10.2\ mm$	
	$A = 115.5\ cm^2$	
	$I_y = 48197\ cm^4$	
	$I_z = 2142\ cm^4$	
	$I_T = 88.57\ cm^4$	
	$I_w = 1236 \times 10^3\ cm^6$	
	$W_{pl,y} = 2194\ cm^3$	
	$W_{pl,z} = 335.9\ cm^3$	
	$W_{el,y} = 1927.9\ cm^3$	
	$W_{el,z} = 214.2\ cm^3$	

Table 2: Results

	SOF.	Ref. [2]
$\overline{\lambda}_y$	0.195	0.195
χ_y	1.0	1.0
$\overline{\lambda}_z$	0.927	0.927
ϕ_z	1.054	1.054
χ_z	0.644	0.644
μ_y	1.000	1.000
μ_z	0.937	0.937

Table 2: (continued)

	SOF.	Ref. [2]
w_y	1.138	1.138
w_z	1.500	1.500
$C_{my,0}$	1.001	0.999
$C_{mz,0}$	0.771	0.771
C_{my}	1.001	1.000
C_{mz}	0.771	0.771
$\overline{\lambda}_0$	0.759	0.757
C_1	1.194	1.200
C_{mLT}	1.139	1.137
$\overline{\lambda}_{LT}$	0.695	0.691
Φ_{LT}	0.825	0.822
χ_{LT}	0.787	0.789
$\chi_{LT,mod}$	0.821	0.826
C_{yy}	0.981	0.981
C_{yz}	0.862	0.863
C_{zy}	0.842	0.843
C_{zz}	1.013	1.014
$nm-y$ (Eq.6.61 [1])	0.966	0.964
$nm-z$ (Eq.6.62 [1])	0.868	0.870

4 Design Process[1]

Design Load:

$M_{z_{Ed}} = 25\ kNm$

$M_{y_{Ed}} = -100\ kNm$ at the start and end of the beam

$M_{y_{Ed}} = 199\ kNm$ at the middle of the beam

$N_{Ed} = 25\ kNm$

Tab. 5.5: Classification of cross-section

The cross-section is classified as **Class** 1, as demonstrated in [2].

6.3.1.2 (1): N_{cr} is the elastic critical force for the relevant buckling mode

$$N_{cr,y} = \frac{\pi^2\ EI_y}{L^2} = 71035\ kN$$

6.3.1.2 (1): $\overline{\lambda}$ non dimensional slenderness for class 1 cross-sections

$$\overline{\lambda}_y = \sqrt{\frac{A\ f_y}{N_{cr,y}}} = 0.195$$

6.3.1.2 (4): $\overline{\lambda} \leq 0.2$ buckling effects may be ignored

$\overline{\lambda}_y < 0.2$ thus $\chi_y = 1.0$

$$N_{cr,z} = \frac{\pi^2\ EI_z}{L^2} = 3157\ kN$$

$$\overline{\lambda}_z = \sqrt{\frac{A\ f_y}{N_{cr,z}}} = 0.927$$

$$\Phi_z = 0.5\left[1 + \alpha_z\left(\overline{\lambda}_z - 0.2\right) + \overline{\lambda}_z^2\right]$$

for rolled I-sections with $h\ /\ b\ >\ 1.2$ and buckling about z-z axis → buckling curve b

6.3.1.2 (2): Table 6.1: Imperfection factors for buckling curves

for buckling curve b → $\alpha_z = 0.34$

$\Phi_z = 1.054$

6.3.1.2 (1): Eq. 6.49: χ_z reduction factor for buckling

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \overline{\lambda}_z^2}} = 0.644 \leq 1.0$$

The stability verification will be done according to Method 1-Annex A of EN 1993-1-1:2005 [1]. Therefore we need to identify the interaction factors according to tables A.1-A.2 of Annex A, EN 1993-1-1:2005 [1].

Auxiliary terms:

Annex A: Tab. A.1: Interaction factors k_{ij} (6.3.3(4)), Auxiliary terms

$$\mu_y = \frac{1 - \dfrac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y\ \dfrac{N_{Ed}}{N_{cr,y}}} = 1.0$$

[1] The sections mentioned in the margins refer to EN 1993-1-1:2005 [1] unless otherwise specified.

$$\mu_z = \frac{1 - \dfrac{N_{Ed}}{N_{cr,z}}}{1 - \chi_z \dfrac{N_{Ed}}{N_{cr,z}}} = 0.937$$

$$w_y = \frac{W_{pl,y}}{W_{el,y}} = 1.138 \leq 1.5$$

$$w_z = \frac{W_{pl,z}}{W_{el,z}} = 1.568 > 1.5 \rightarrow w_z = 1.5$$

Determination of $C_{mi,0}$ factors

The general formula for combined end moments and transverse loads is used here.

$$C_{my,0} = 1 + \left(\frac{\pi^2 \, EI_y \, |\delta_z|}{L^2 \, \left| M_{y,Ed,right} \right|} - 1 \right) \frac{N_{Ed}}{N_{cr,y}}$$

Annex A: Tab. A.2: Equivalent uniform moment factors $C_{mi,0}$

$$\delta_z = 3.33 \; mm$$

$$C_{my,0} = 1.001$$

The formula for linearly distributed bending moments is used here.

$$\psi_z = \frac{M_{z,ED,right}}{M_{z,Ed,left}} = 0/25 = 0$$

$$C_{mz,0} = 0.79 + 0.21\psi_z + 0.36(\psi_z - 0.33)\frac{N_{Ed}}{N_{cr,z}} = 0.771$$

$$C_{mz} = C_{mz,0} = 0.771$$

Resistance to lateral torsional buckling

Because $I_T = 8.857 \times 10{-}7m^4 < I_y = 4.820 \times 10{-}4m^4$, the cross-section shape is such that the member may be prone to lateral torsional buckling.

The support conditions of the member are assumed to be the so-called "fork conditions", thus $L_{LT} = L$.

$$M_{cr,0} = \sqrt{\frac{\pi^2 EI_z}{L_{LT}^2} \left(GI_T + \frac{\pi^2 EI_w}{L_{LT}^2} \right)} = 895 kNm$$

$$\overline{\lambda}_0 = \sqrt{\frac{W_{pl,y} \, f_y}{M_{cr,0}}} = 0.759$$

Annex A: Tab. A.1: $\overline{\lambda}_0$: non-dimensional slenderness for lateral-torsional buckling due to uniform bending moment

$$N_{cr,T} = \frac{A}{I_y + I_z} \left(GI_T + \frac{\pi^2 EI_w}{L_{LT}^2} \right) = 5822 kN$$

$C_1 = 1.194$ determined by eigenvalue analysis

$$\overline{\lambda}_{0lim} = 0.2\sqrt{C_1} \sqrt[4]{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)\left(1 - \frac{N_{Ed}}{N_{cr,T}}\right)} = 0.205$$

Annex A: Tab. A.1: Auxiliary terms

$$\overline{\lambda}_0 = 0.759 >= \overline{\lambda}_{0lim} = 0.205$$

Lateral torsional buckling has to be taken into account.

Annex A: Tab. A.1: Auxiliary terms

$$a_{LT} = 1 - \frac{I_T}{I_y} = 0.998 >= 0$$

Annex A: Tab. A.1: ϵ_y for class 1 cross-section

$$\epsilon_y = \frac{M_{y,Ed,right}}{N_{Ed}} \frac{A}{W_{el,y}} = 2.383$$

$$C_{my} = C_{my,0} + \left(1 - C_{my,0}\right) \frac{\alpha_{LT}\sqrt{\epsilon_y}}{1 + \alpha_{LT}\sqrt{\epsilon_y}} = 1.001$$

$$C_{mLT} = C_{my}^2 \frac{\alpha_{LT}}{\sqrt{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)\left(1 - \frac{N_{Ed}}{N_{cr,T}}\right)}} = 1.139 \geq 1.0$$

thus $C_{mLT} = 1.0$.

6.3.2.2: Lateral torsional buckling curves - General case

The general case method is chosen here.

$M_{cr} = 1068\ kNm$, determined by eigenvalue analysis

6.3.2.2 (1): $\overline{\lambda}_{LT}$ non dimensional slenderness for lateral torsional buckling

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_{pl,y}\, f_y}{M_{cr}}} = \sqrt{\frac{2194 \cdot 10^{-6} \cdot 235 \cdot 10^6}{1079 \cdot 10^3}} = 0.695$$

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^2\right]$$

6.3.2.2 (1): Table 6.4: Recommendation for the selection of ltb curve for cross-sections using Eq. 6.56

for rolled I-sections and $h\,/\,b > 2 \rightarrow$ buckling curve b

6.3.2.2 (2): Table 6.4: Recommendation values for imperfection factors α_{LT} for ltb curves

for buckling curve b $\rightarrow \alpha_{LT} = 0.34$

$$\Phi_{LT} = 0.825$$

6.3.2.2 (1): Eq. 6.56: χ_{LT} reduction factor for ltb

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \overline{\lambda}_{LT}^2}}$$

$$\chi_{LT} = 0.787 \leq 1.0$$

$k_c = 0.907$ determined by eigenvalue analysis through the C_1 factor.

6.3.2.3 (2): For taking into account the moment distribution between the lateral restraints of members the reduction factor χ_{LT} may be modified

$$f = 1 - 0.5\,(1 - k_c)\left[1 - 2\left(\overline{\lambda}_{LT} - 0.8\right)^2\right] = 0.959 \leq 1.0$$

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} = 0.821 \leq 1.0$$

Elastic-plastic bending resistances

$$N_{c,Rk} = A \cdot f_y = 2715\ kN$$

$$M_{pl,y,Rk} = W_{pl,y} \cdot f_y = 516\ kNm$$

$$M_{pl,z,Rk} = W_{pl,z} \cdot f_y = 78.9\ kNm$$

$$\overline{\lambda}_{max} = \overline{\lambda}_z = 0.927$$

Annex A: Tab. A.1: Auxiliary terms: $\overline{\lambda}_{max} = max(\overline{\lambda}_y, \overline{\lambda}_z)$

$$b_{LT} = 0.5 \cdot \alpha_{LT} \cdot \overline{\lambda}_0^2 \frac{M_{y,Ed}}{\chi_{LT,mod} \frac{M_{pl,y,Rk}}{\gamma_{M1}}} \frac{M_{z,Ed}}{\frac{M_{pl,z,Rk}}{\gamma_{M1}}} = 0.428$$

Annex A: Tab. A.1: Auxiliary terms

$$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1.6}{w_y} \cdot C_{my}^2 \overline{\lambda}_{max} - \frac{1.6}{w_y} \cdot C_{my}^2 \overline{\lambda}_{max}^2\right) \cdot \frac{N_{Ed}}{\frac{N_{c,Rk}}{\gamma_{M1}}} - b_{LT}\right]$$

$$C_{yy} = 0.981 \geq \frac{W_{el,y}}{W_{pl,y}} = 0.879$$

$$c_{LT} = 10 \cdot \alpha_{LT} \cdot \frac{\overline{\lambda}_0^2}{5 + \overline{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \cdot \chi_{LT,mod} \frac{M_{pl,y,Rk}}{\gamma_{M1}}} = 0.471$$

$$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{my}^2 \overline{\lambda}_{max}^2}{w_z^5}\right) \frac{N_{Ed}}{\frac{N_{c,Rk}}{\gamma_{M1}}} - c_{LT}\right]$$

$$C_{yz} = 0.862 \geq 0.6 \sqrt{\frac{w_z}{w_y} \frac{W_{el,z}}{W_{pl,z}}} = 0.439$$

$$d_{LT} = 2 \cdot \alpha_{LT} \cdot \frac{\overline{\lambda}_0}{0.1 + \overline{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \cdot \chi_{LT,mod} \frac{M_{pl,y,Rk}}{\gamma_{M1}}} \frac{M_{z,Ed}}{C_{mz} \frac{M_{pl,z,Rk}}{\gamma_{M1}}} = 0.348$$

$$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \overline{\lambda}_{max}^2}{w_y^5}\right) \frac{N_{Ed}}{\frac{N_{c,Rk}}{\gamma_{M1}}} - d_{LT}\right]$$

$$C_{zy} = 0.842 \geq 0.6 \sqrt{\frac{w_y}{w_z} \frac{W_{el,y}}{W_{pl,y}}} = 0.459$$

$$e_{LT} = 1.7 \cdot \alpha_{LT} \cdot \frac{\overline{\lambda}_0}{0.1 + \overline{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \cdot \chi_{LT,mod} \frac{M_{pl,y,Rk}}{\gamma_{M1}}} = 0.721$$

$$C_{zz} = 1 + (w_z - 1)\left[\left(2 - \frac{1.6}{w_z} \cdot C_{mz}^2 \overline{\lambda}_{max} - \frac{1.6}{w_z} \cdot C_{mz}^2 \overline{\lambda}_{max}^2 - e_{LT}\right) \cdot \frac{N_{Ed}}{\frac{N_{c,Rk}}{\gamma_{M1}}}\right]$$

$$C_{zz} = 1.013 \geq \frac{W_{el,z}}{W_{pl,z}} = 0.667$$

Verification

6.3.3 Uniform members in bending and axial compression

According to 1993-1-1:2005, 6.3.3 (4), members which are subjected to combined bending and axial compression should satisfy:

$$\frac{N_{Ed}}{\chi_y \frac{N_{c,Rk}}{\gamma_{M1}}} + \mu_y \left[\frac{C_{mLT}}{\chi_{LT,mod}} \frac{C_{my} \cdot M_{y,Ed}}{\left(1 - \frac{N_{Ed}}{N_{cr,y}}\right) C_{yy} \cdot \frac{M_{pl,y,Rk}}{\gamma_{M1}}} \right.$$
$$\left. +0.6 \sqrt{\frac{w_z}{w_y}} \frac{C_{mz}\, M_{z,Ed}}{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right) C_{yz} \frac{M_{pl,z,Rk}}{\gamma_{M1}}} \right]$$

6.3.3 (4): Eq. 6.61: Members which are subjected to combined bending and axial compression should satisfy:

$$\frac{500}{1.0 \frac{2715}{1.0}} + 1.0 \left[\frac{1.139}{0.821} \frac{1.001 \cdot 198.9}{\left(1 - \frac{500}{71035}\right) 0.981 \frac{516}{1.0}} \right.$$
$$\left. +0.6 \sqrt{\frac{1.500}{1.138}} \frac{0.771 \cdot 25}{\left(1 - \frac{500}{3157}\right) 0.862 \frac{78.9}{1.0}} \right]$$
$$= 0.966 \leq 1.00$$

→ Satisfactory

$$\frac{N_{Ed}}{\chi_z \frac{N_{c,Rk}}{\gamma_{M1}}} + \mu_z \left[0.6 \sqrt{\frac{w_y}{w_z}} \frac{C_{mLT}}{\chi_{LT,mod}} \frac{C_{my} \cdot M_{y,Ed}}{\left(1 - \frac{N_{Ed}}{N_{cr,y}}\right) C_{zy} \frac{M_{pl,y,Rk}}{\gamma_{M1}}} \right.$$
$$\left. + \frac{C_{mz}\, M_{z,Ed}}{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right) C_{zz} \frac{M_{pl,z,Rk}}{\gamma_{M1}}} \right]$$

6.3.3 (4): Eq. 6.62: Members which are subjected to combined bending and axial compression should satisfy:

$$\frac{500}{0.644 \frac{2715}{1.0}} + 0.937 \left[+0.6 \sqrt{\frac{1.138}{1.500}} \frac{1.139}{0.821} \right.$$
$$\left. \cdot \frac{1.001 \cdot 198.9}{\left(1 - \frac{500}{71035}\right) 0.842 \frac{516}{1.0}} + \frac{0.771 \cdot 25}{\left(1 - \frac{500}{3157}\right) 1.013 \frac{78.9}{1.0}} \right]$$

$$= 0.868 \leq 1$$

$\rightarrow$Satisfactory

5 Conclusion

This example shows the ckeck for lateral torsional buckling of steel members. The small deviations that occur in some results come from the fact that there are some small differences in the sectional values between SOFiSTiK and the reference solution. Therefore, these deviations are of no interest for the specific verification process. In conclusion, it has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *EN 1993-1-1: Eurocode 3: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2005.

[2] ECCS Technical Committee 8 Stability. *Rules for Member Stability in EN 1993-1-1, Background documentation and design guidelines*. Tech. rep. No 119. European Convention for Constructional Steelwork (ECCS), 2006.