Benchmark Example No. 51

Pushover Analysis: Performance Point Calculation by EC8 Procedure

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VERiFiCATiON
BE51 Pushover Analysis: Performance Point Calculation by EC8 Procedure

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Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview	
Element Type(s):	
Analysis Type(s):	
Procedure(s):	
Topic(s):	EQKE
Module(s):	SOFiLOAD
Input file(s):	pushover-pp-ec8.dat

1   Problem Description

The following example is intended to verify the Eurocode 8 (EC8) procedure for the calculation of the performance point (illustrated schematically in Fig. 1), as implemented in SOFiSTiK. The elastic demand and capacity diagrams are assumed to be know.
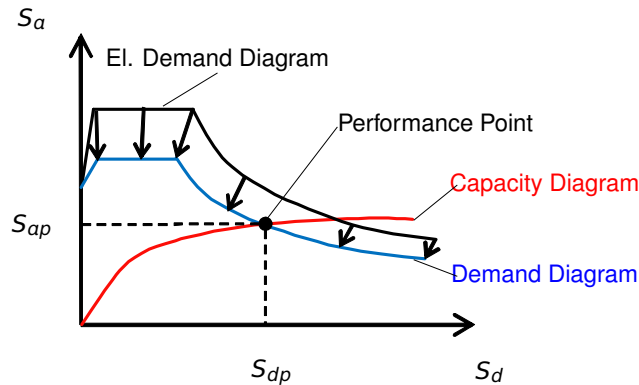

Figure 1: Determination of the performance point PP (S_{dp}, S_{ap})

2   Reference Solution

The reference solution is provided in [1].

Assuming that the elastic demand diagram (5% elastic response spectrum in ADRS format[1]) and the capacity diagram are known, it is possible to determine the performance point PP (S_{dp}, S_{ap}) (Fig. 1). The procedure comprises of a series of trial calculations (trial performance points PP_t ($S_{dp,t}, S_{ap,t}$)), in which the equivalent *inelastic* single degree of freedom system (SDOF), represented by the capacity diagram, is idealized with the equivalent inelastic SDOF system with a bi-linear force-deformation relationship. The response in form of the performance point PP is then calculated from the *inelastic* response spectrum (demand diagram). The computation stops when the performance point PP is within a tolerance of a trial performance point PP_t. Detailed description of this procedure can be found in [2], [3], [1] and [4].

In the reference example [1] the bi-linear idealization of the capacity is assumed to be independent of the performance point and it is performed at the beginning of the analysis. This eliminates the need for the iterations and the solution of the problem can be obtained in a single calculation step.

Hence in this example it is assumed that the bi-linear idealization of the capacity diagram is already known, which means that the point PY (S_{dy}, S_{ay}) is given. The procedure to calculated the performance point is illustrated in Fig. 2 and can be summarized as follows [4]:

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[1]ADRS = Spectral Acceleration S_a - Spectral Displacement S_d format

(a) Short period range, $T^* < T_C$ (b) Medium and long range, $T^* > T_C$
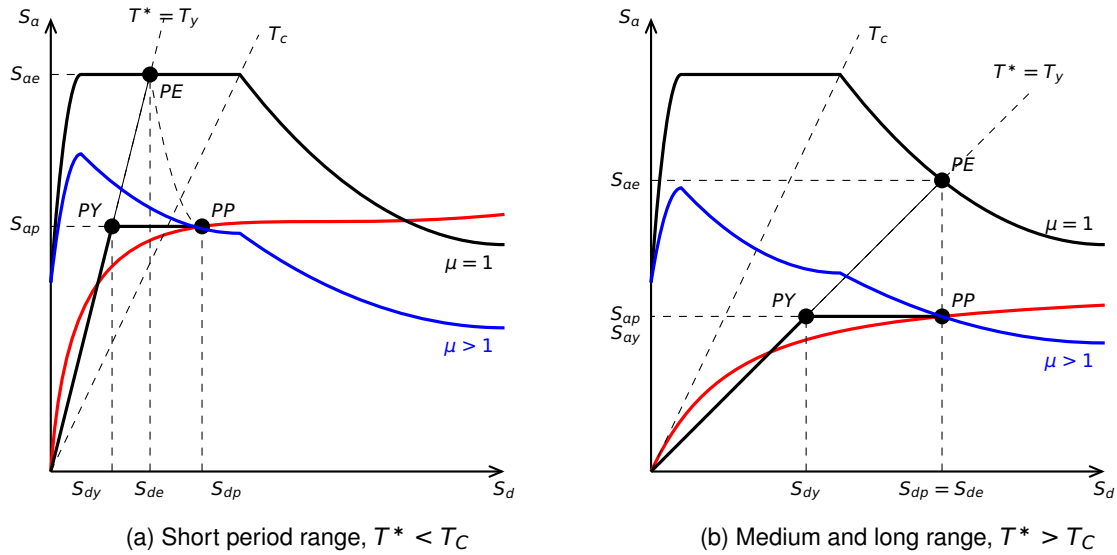
Figure 2: Determination of the performance point PP for the equivalent SDOF system

1. Determine the period of the idealized system $T^* = T_y$ from the known PY (S_{dy}, S_{ay}):

$$T^* = T_y = 2\pi \cdot \sqrt{\frac{S_{dy}}{S_{ay}}} \; ; \tag{1}$$

2. Calculate the elastic spectral response PE (S_{de}, S_{ae}) of the idealized equivalent SDOF system with the period $T^* = T_y$ from the given 5%-damped elastic response spectrum (Fig. 2);

3. Calculate the yield strength reduction factor R_y:

$$R_y = \frac{S_{ae}}{S_{ay}} \; ; \tag{2}$$

4. Calculate ductility μ:

$$\mu = \begin{cases} (R_y - 1) \cdot \dfrac{T_C}{T^*} + 1 & \text{for} \quad T^* < T_C \\ R_y & \text{for} \quad T^* \geq T_C \end{cases} \; ; \tag{3}$$

5. Determine the performance point PP (S_{dp}, S_{ap}) from the inelastic design spectrum:

$$S_{dp} = \mu \cdot S_{dy} = \mu \cdot \frac{S_{de}}{R_y} \; , \tag{4a}$$

$$S_{ap} = \frac{S_{ae}(T^*)}{R_y} \; . \tag{4b}$$

3 Model and Results

In order to verify the analysis procedure for the determination of the performance point, a test case has been set up in such a way that it comprises of a SDOF with a unit mass and a non-linear spring element. It is obvious that for such an element the quantities governing the transformation from the original system

to the equivalent inelastic SDOF system must be equal to one, i.e.

$$\phi_{cnod} = 1 \quad ; \quad \Gamma = 1 \quad ; \quad m = 1 \;, \tag{5}$$

where ϕ_{cnod} is the eigenvector value at control node, Γ is the modal participation factor and m is the generalized modal mass. Writing now the equations which govern the conversion of the pushover curve to capacity diagram, we obtain [4]

$$S_d = \frac{u_{cnod}}{\phi_{cnod} \cdot \Gamma} = u_{cnod} \;, \tag{6a}$$

$$S_a = \frac{V_b}{\Gamma^2 \cdot m} = V_b \;, \tag{6b}$$

where V_b is the base shear and u_{cnod} is the control node displacement.

Since the original system is a SDOF system, V_b and u_{cnod} are nothing else but the force in spring P and the displacement of the unit mass u, respectively. It follows further that the force-displacement work law assigned to the spring element corresponds to the capacity diagram in ADRS format, with the force P and displacement u equal to S_a and S_d, respectively.

The bi-linear idealization of the capacity diagram used in the reference example is defined by two points, whose coordinates are listed in the Table 1 [2]. According to the analysis above, these points can be used to define the force- displacement work law $P - u$ of the non-linear spring element (Fig. 3).

Table 1: Model Properties [1]

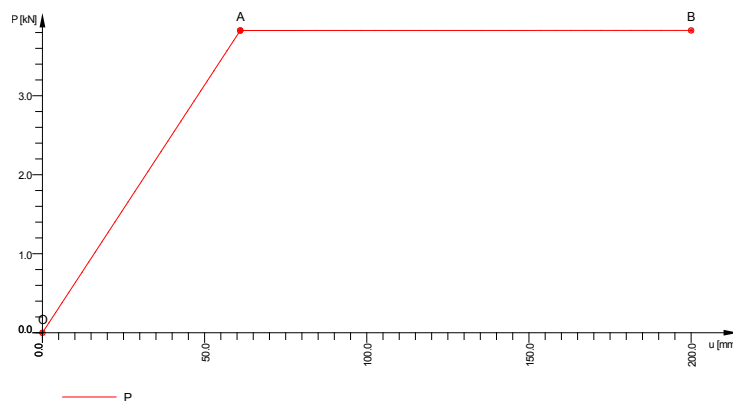| Capacity Diagram		Elastic Demand
Point	$(S_d[mm], S_a[m/s^2])$	5%-Damped Elastic Response Spectrum
A	$(61, 3.83)$	$a_g = \{0.60g, 0.30g, 0.16g\}$
B	$(\infty, 3.83)$	$S_A = 1.0, S_B = 2.5, k_1 = 1.0$
		$T_B = 0.15s, T_C = 0.60s, T_D = 3.00s$

Figure 3: Force-displacement work law of the non-linear spring

The elastic demand is a 5%-damped elastic response spectrum, whose properties are summarized in Table 1. Three levels of peak ground acceleration a_g have been taken into an account. The shape of

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[2]Not that the point A is nothing else but the point PY (S_{dy}, S_{ay}).

the spectrum and the meaning of the parameters specified in Table 1 are shown in Figure 4.

$$0 \leq T \leq T_B:$$
$$S = S_A + \frac{T}{T_B} \cdot (S_B - S_A)$$

$$T_B \leq T \leq T_C:$$
$$S = S_B$$

$$T_C \leq T \leq T_D:$$
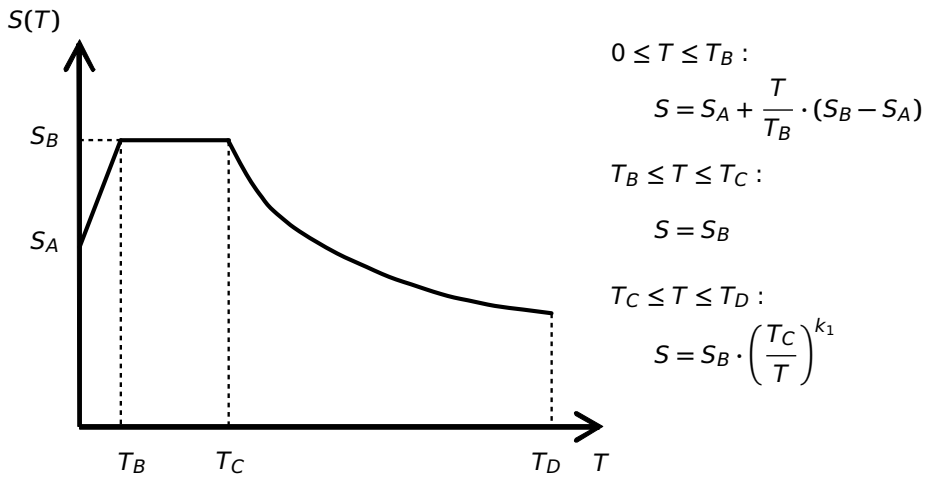$$S = S_B \cdot \left(\frac{T_C}{T}\right)^{k_1}$$

Figure 4: 5%-Damped Elastic Response Spectrum (El. Demand Diagram)

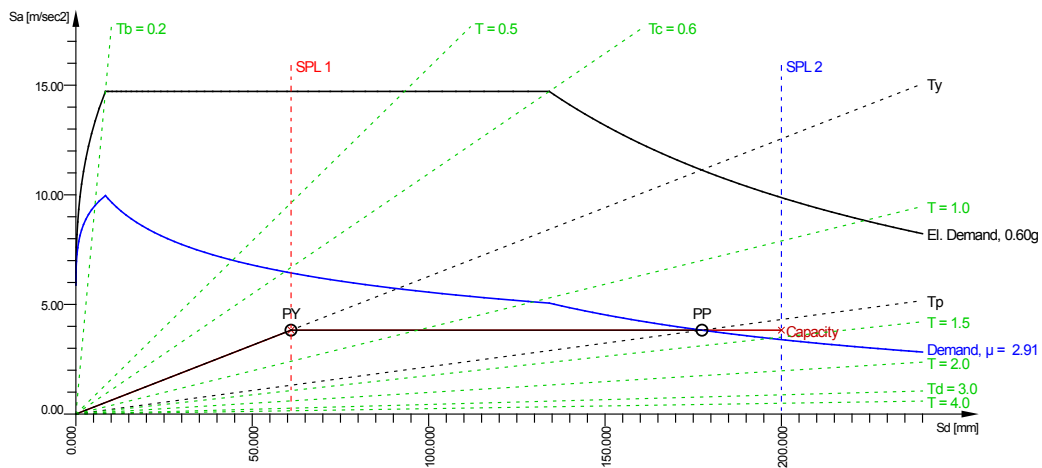The outcome of the analysis is shown in Figures 5 to 7.

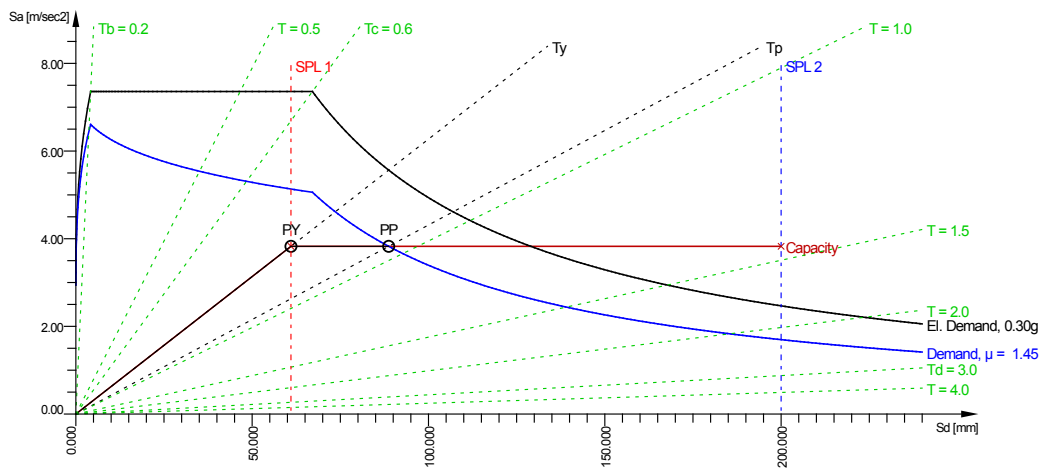Figure 5: Capacity-Demand-Diagram ($a_g = 0.60g$)

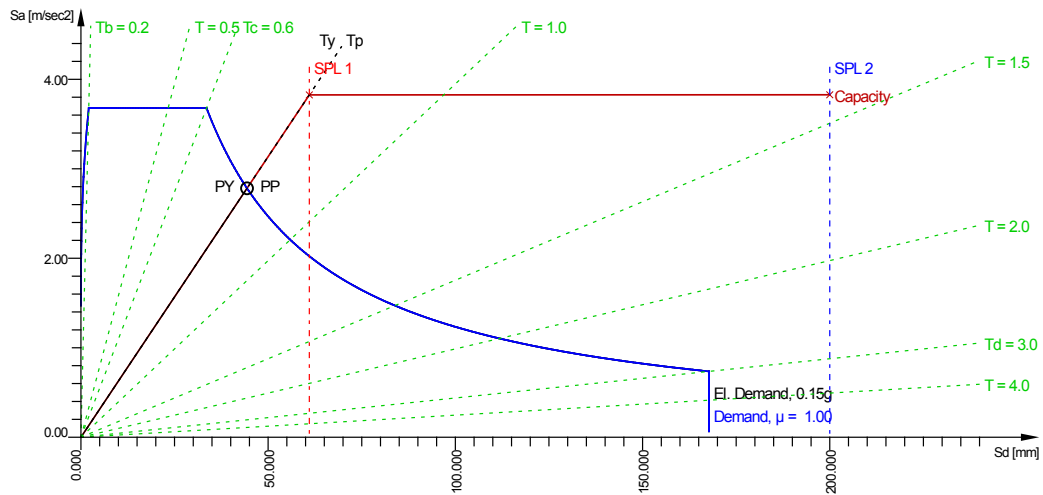Figure 6: Capacity-Demand-Diagram ($a_g = 0.30g$)

Figure 7: Capacity-Demand-Diagram ($a_g = 0.15g$)

The results of the SOFiSTiK calculation and the comparison with the reference solution are summarized in Table 2.

Table 2: Results

a_g		μ	R_{yp}	T_y	S_{dy}	S_{dp}	S_{ap}
[g]		[−]	[−]	[s]	[mm]	[mm]	[m/s²]
	SOF.	2.9	2.9	0.79	61	177	3.83
0.60	Ref. [1]	2.9	2.9	0.79	61	177	3.83
	|e| [%]	0.0	0.0	0.0	0.0	0.0	0.0
	SOF.	1.5	1.5	0.79	61	89	3.83
0.30	Ref. [1]	1.5	1.5	0.79	61	89	3.83
	|e| [%]	0.0	0.0	0.0	0.0	0.0	0.0
	SOF.	1.0	1.0	0.79	44	44	2.78
0.15	Ref. [1]	1.0	1.0	0.79	44	44	2.76
	|e| [%]	0.0	0.0	0.0	0.0	0.0	0.7

μ	displacement ductility factor
R_{yp}	reduction factor due to ductility at performance point
T_y	period associated with yielding point
S_{dy}, S_{dp}	spectral displacements at yielding and performance point
S_{ap}	pseudo spectral acceleration at performance point

The results are in excellent agreement with the reference solution.

4 Conclusion

Excellent agreement between the reference and the results computed by SOFiSTiK verifies that the procedure for the calculation of the performance point according to Eurocode 8 is adequately implemented.

5 Literature

[1] P. Fajfar. "A Nonlinear Analysis Method for Performance-Based Seismic Design". In: *Earthquake Spectra* 16.3 (2000), pp. 573–592.

[2] EN1998-1:2004. *Eurocode 8: Design of structures for earthquake resistance, Part 1: General rules, seismic actions and rules for buildings*. CEN. 2004.

[3] P. Fajfar. "Capacity Spectrum Method Based on Inelastic Demand Spectra". In: *Earthquake engineering and structural dynamics* 28.9 (1999), pp. 979–993.

[4] *SOFiLOAD Manual: Loads and Load Functions.* Version 2018-0. SOFiSTiK AG. Oberschleißheim, Germany, 2017.