Benchmark Example No. 42

Thick Circular Plate

SOFiSTiK | 2022

VERiFiCATiON
BE42 Thick Circular Plate

VERiFiCATiON Manual, Service Pack 2022-12 Build 74

SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview

Element Type(s): C3D
Analysis Type(s): STAT
Procedure(s):
Topic(s):
Module(s): ASE
Input file(s): thick_plate.dat

1 Problem Description

The problem consists of a circular plate with a constant area load, as shown in Fig. 1. The system is modelled as a plane problem and the deflection in the middle of the plate is determined for various thicknesses [1].

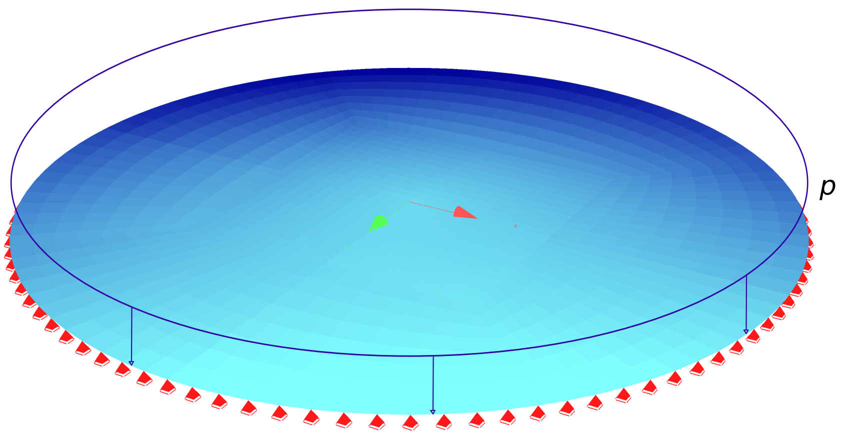

Figure 1: Problem Description

2 Reference Solution

Depending on the various thicknesses of the plate, the maximum deflection w in the middle of the plate can be obtained as $w = w_B + w_S$, where w_B is the dislacement due to bending and w_S is the displacement due to shear strains, determined as follows [2]:

$$w_B = \frac{p \cdot r^4}{64 \cdot K} \frac{(5+\mu)}{(5+\mu)} \tag{1}$$

$$K = \frac{E \cdot h^3}{12(1-\mu^2)} \tag{2}$$

$$w_S = \frac{1.2 \cdot p \cdot r^2}{4 \cdot G \cdot h} \tag{3}$$

where p is the load ordinate, r the radius, E the elasticity modulus, h the plate thickness, μ the Poissons ratio and G the shear modulus.

The maximum bending moment at the middle of the plate is independent of the plate thickness and corresponds for the specific load case to

$$M_x = M_y = \frac{p \cdot r^2}{16} \cdot (3 + \mu) = 4928.125 \qquad [kNm/m] \tag{4}$$

3 Model and Results

The properties of the model are defined in Table 1. The plate is modelled as a plane system with three degrees of freedom, u_z, ϕ_x, ϕ_y, per node and u_z hinged at the edge, as shown in Fig. 1. The weight of the system is not considered. A constant area load $p = 1000\ kN/m^2$ is applied, as shown in Fig. 1. The system is modelled with 1680 quadrilateral elements, as presented in Fig. 2, and a linear analysis is performed for increasing thicknesses. The results are presented in Table 2 where they are compared to the analytical solution calculated from the formulas presented in Section 2 and the influence of the varying thickness is assesed.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 3000\ kN/cm^2$	$h = 0.5 - 2.5\ m$	$p = 1000\ kN/m^2$
$G = 1300\ kN/cm^2$	$r = 5\ m$	
$\mu = 0.154$	$D = 10\ m$	

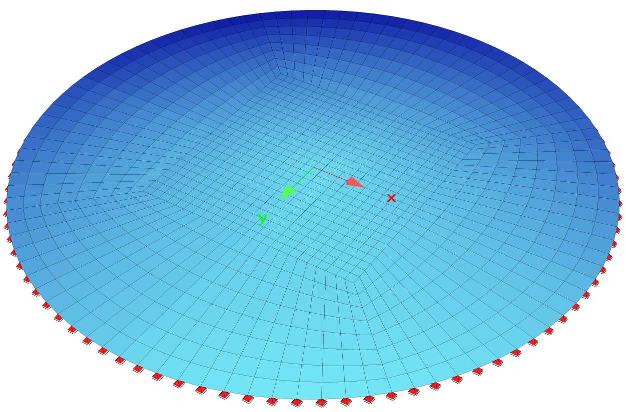

Figure 2: FEM model

The maximum bending moment is calculated at the middle of the plate, as $M_x = M_y = 4932.244$ $[kNm/m]$ with a deviation of 0.08 %.

Table 2: Results

h $[m]$	h/D	Analytical u_z $[mm]$	SOF. u_z $[mm]$	$\lvert e_r \rvert$ [%]
0.50	0.05	137.413	137.440	0.02
1.00	0.10	17.609	17.618	0.05
1.50	0.15	5.431	5.437	0.11
2.00	0.20	2.418	2.421	0.14
2.50	0.25	1.321	1.324	0.23

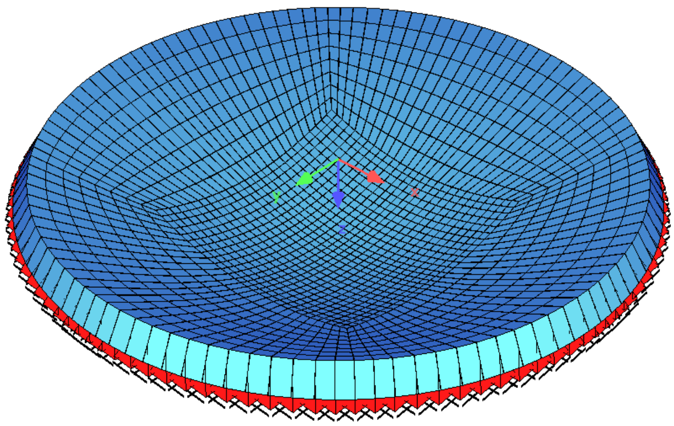

Figure 3: Displacements

4 Conclusion

The example allows the verification of the calculation of thick plates. It has been shown, that the calculated results are in very good agreement with the analytical solution even for thicker plates.

5 Literature

[1] *VDI 6201 Beispiel: Softwaregestütze Tragwerksberechnung - Beispiel Dicke Platte, Kategorie 1: Mechanische Grundlagen*. Verein Deutscher Ingenieure e. V.

[2] F. U. Mathiak. *Ebene Flächentragwerke Teil II, Grundlagen der Plattentheorie*. Hochschule Neubrandenburg. 2011.