Benchmark Example No. 41

Linear Pinched Cylinder

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VERiFiCATiON
BE41 Linear Pinched Cylinder

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Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview	
Element Type(s):	C3D
Analysis Type(s):	STAT
Procedure(s):	
Topic(s):	
Module(s):	ASE
Input file(s):	cylinder.dat

1 Problem Description

The problem consists of a thin cylinder shell with rigid end diaphragms, which is loaded in its middle by two oppositely directed radially point loads, as shown in Fig 1. The maximum deflection at the center of the cylinder, under the point loads, is determined and verified for refined meshes [1].

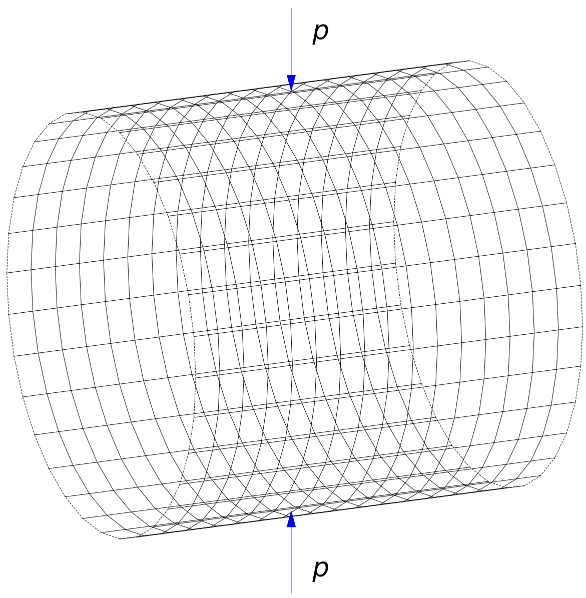

Figure 1: Problem Description

2 Reference Solution

There is a convergent numerical solution of $w = 1.8248 \cdot 10^{-5}$ for the radial displacement at the loaded points, as given by Belytschko [2]. This problem is one of the most severe tests for both inextensional bending and complex membrane states of stress [3] .

3 Model and Results

The properties of the model are defined in Table 1. The geometric parameters and the material are all dimensionless. The compressive point load $p = 1.0$ is applied radially and in opposite directions at the middle nodes of the cylinder, as shown in Fig. 1. Using symmetry, only one-eighth of the cylinder needs to be modeled, as shown in Fig. 2. For the simplified model only one fourth of the load $p*$ is applied at the the upper middle node, as it can be visualised in Fig. 2. The end of the cylinder is supported by a rigid diaphragm [4], while at the two edges of the cylinder, parallel to the x- and y- axis, symmetry support conditions are employed. In the plane of middle of the cylinder, the displacements in the longitudial direction, as well as the rotations around x- and y- axis are fixed. The example allows the verification of the calculation of thin shells with increasingly refined regular meshes.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 3.0 \cdot 10^6\ MPa$	$L = 600$, $l = 300$	$p = 1.0$
$\mu = 0.30$	$r = 300$	$p* = 0.25$
	$t = 3$	

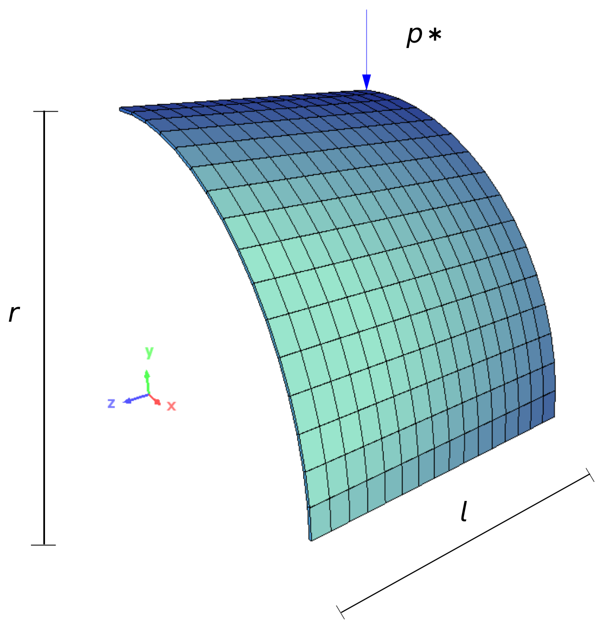

Figure 2: FEM model

Table 2: Normalised Point-Load Displacement w/w_a with Mesh Refinement

Element/Side	Conforming Element	Non-Conforming Element
4	0.4525	0.5917
8	0.8214	0.9057
16	0.9701	1.0082

The results are presented in Fig 3 and Table 2, where they are compared to the analytical solution as presented in Section 2. Two element formulations are considered. The first one, represented by the red curve, corresponds to the 4-node regular conforming element whereas the second, represented by the purple curve corresponds to the non-conforming element with six functions, offering a substantial improvement of the results.

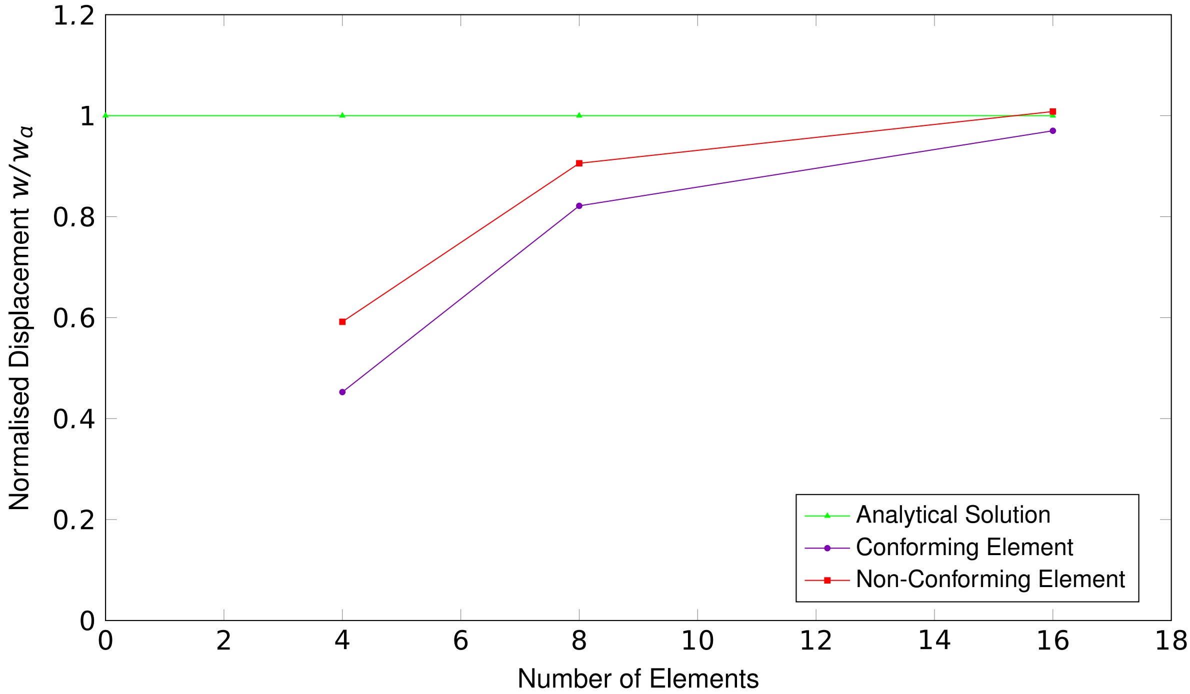

Figure 3: Convergence Diagram

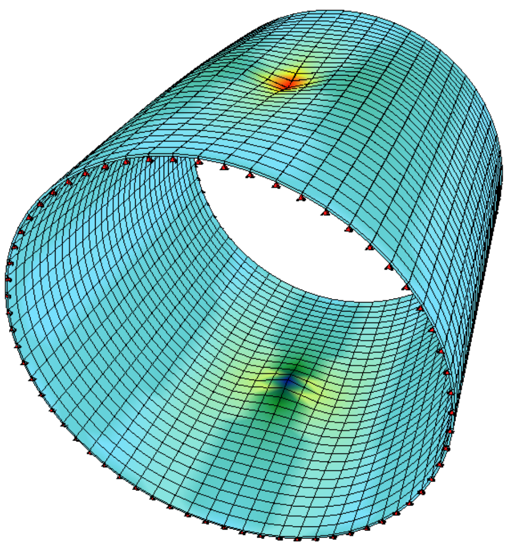

Figure 4: Deformed Shape

4 Conclusion

The example allows the verification of the calculation of thin shells. For increasing refined meshes, the calculated result for both types of elements convergence fast to the predetermined analytical solution. The advantage of the utilisation of the non-conforming element is evident, since it is in excellent agreement with the analytical solution for a refined mesh.

5 Literature

[1] *VDI 6201 Beispiel: Softwaregestütze Tragwerksberechnung - Beispiel Zylinderschale mit starren Endscheiben, Kategorie 1: Mechanische Grundlagen*. Verein Deutscher Ingenieure e. V.

[2] T. Belytschko et al. “Stress Projection for Membrane and Shear Locking in Shell Finite Elements”. In: *Computer Methods in Applied Mechanics and Engineering* 53(1-3) (1985), pp. 221–258.

[3] T. Rabczuk, P. M. A. Areias, and T. Belytschko. “A meshfree thin shell method for non-linear dynamic fracture”. In: *International Journal for Numerical Methods in Engineering* 72(5) (2007), pp. 524–548.

[4] P. Krysl and T. Belytschko. “Analysis of thin shells by the element-free Galerkin method”. In: *International Journal for Solids and Structures* 33(20-22) (1996), pp. 3057–3080.