Benchmark Example No. 37

Beam Calculation of Varying Cross-Section according to Second Order Theory

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VERiFiCATiON
BE37 Beam Calculation of Varying Cross-Section according to Second Order Theory

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Front Cover

Arnulfsteg, Munich Photo: Hans Gössing

Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT, GNL
Procedure(s):	STAB
Topic(s):	
Module(s):	ASE, STAR2, DYNA
Input file(s):	beam_th2.dat

1 Problem Description

The problem consists of a column of continuously varying cross-section, subjected to a large compressive force in combination with imperfections as well as horizontal and temperature loads, as shown in Fig. 1. The forces and deflections of the structure, calculated according to second order theory, are determined.

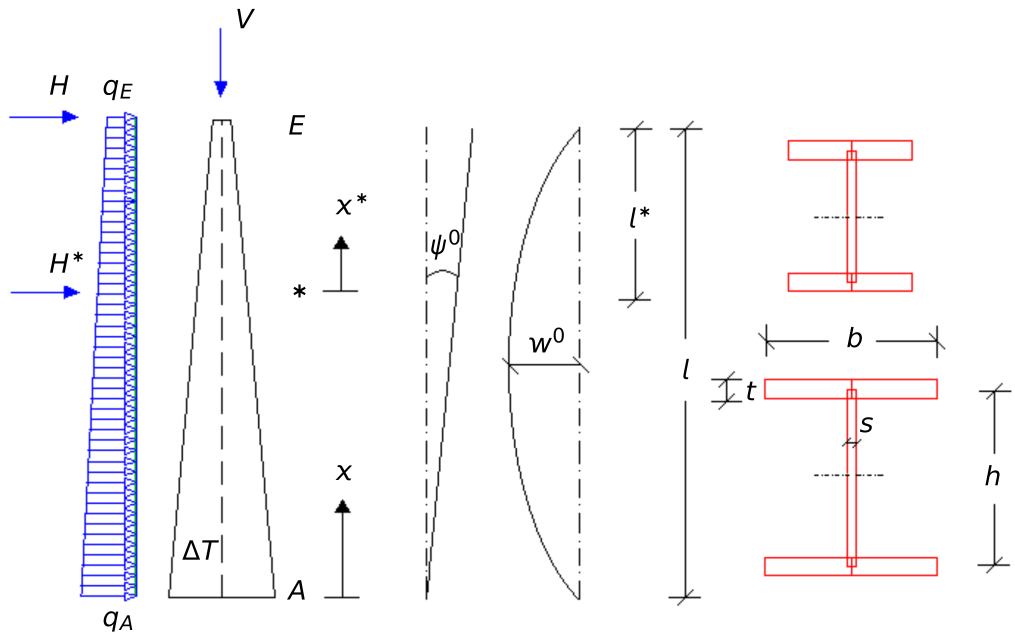

Figure 1: Problem Description

2 Reference Solution

This example attempts to give a complete description of the forces and the deflections of a beam with varying cross-section calculated with second order theory. As a reference solution, a general formulation concept is adopted, where through the application of series functions, uniform formulas can be derived to describe the beam behaviour of varying cross-section. In this concept, the cross-section properties can vary according to a polynomial of arbitrary degree, the normal force, with respect to second order theory, is assumed constant, the imperfections or predeformations as well as the temperature loads are taken into account and the deformations due to moments and normal forces are treated. Further information on the reference solution can be found in Rubin (1991) [1].

3 Model and Results

The general properties of the model [1] are defined in Table 1 and the cross-sections in Table 2. A general linear material is used and a linearly varying, thin-walled I-beam profile for the cross-section.

The shear deformations are neglected. A safety factor of 1.35 is used for the dead weight, giving a total normal force of $N = -0.5 - 0.0203 = -0.5203\ MN$. An imperfection of linear distribution with maximum value of 60 mm at node E is applied, as well as one of quadratic distribution with maximum value of $-48\ mm$ at the middle. The temperature load is given as a difference of temperature of $25°C$, between the left and the right side of the beam. The height of the cross-section is taken as the height of the web only. Second order theory is utilised and the structure is analysed both with ASE and STAR2.

Table 1: Model Properties

Model Properties	Loading
$E = 21\ MN/cm^2$, $\gamma_g = 1.35$	$V = 500\ kN$
$\psi^0 = 1/200$, $w^0 = -48\ mm$	$q_E = 6\ kN/m$, $q_A = 10\ kN/m$
$\alpha_T = 1.2 \times 10^{-5}\ 1/°K$	$\Delta T = T_{right} - T_{left} = -25\ °C$
$l = 12\ m$, $l^* = 4\ m$	$H = 20\ kN$, $H^* = 10\ kN$

Table 2: Cross-sectional Properties

Position	Web $[mm]$		Flange $[mm]$		Area $[cm^2]$	I_y $[cm^4]$
	h	s	b	t		
E	200	12	194	20	101.6	8560
$*$	300	12	260.7	20	140.27	26160.3
A	500	12	394	20	217.6	111000

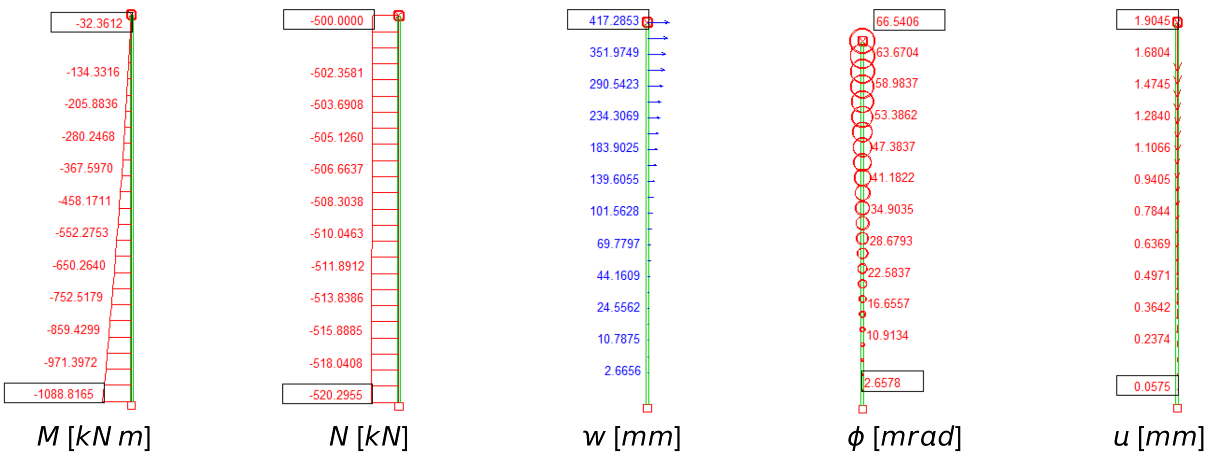

Figure 2: Results with Twenty Four Beam Elements calculated by ASE

The results are presented in Table 3, where they are compared to the reference solution according to Rubin (1991) [1]. Fig. 2 shows the forces and deflections of the structure as they have been calculated by ASE with twenty four beam elements.

Table 3: Results

Number of Elements		ϕ_E [$mrad$]	w_E [mm]	u_E [mm]	M_A [$MN\,m$]	N_A [MN]	N_E [MN]	N_Ki [MN]
-	Ref. [1]	67.70	423.50	−1.9050	−1.10	−0.5203	−0.50	−1.882
3	ASE	63.8819	402.8733	−1.8937	−1.0816	−0.5202	−0.50	−1.9556
	STAR2	82.1398	508.5219	−1.9288	−1.1352	−0.5203	−0.50	-
6	ASE	65.8787	413.7793	−1.9018	−1.0871	−0.5203	−0.50	−1.8993
	STAR2	70.0395	437.2366	−1.9108	−1.0990	−0.5203	−0.50	-
24	ASE	66.5406	417.2853	−1.9045	−1.0888	−0.5203	−0.50	−1.8827
	STAR2	66.7933	418.7001	−1.9050	−1.0895	−0.5203	−0.50	-

4 Conclusion

This example examines the behaviour of a tapered beam, treated with second order theory. The results, calculated both with ASE and STAR2, converge to the same solution as the number of elements increases. Their deviation arises from the fact that ASE uses an exponential interpolation based on area and inertia as well as numerical integration of the stiffness, while STAR2 uses the geometric mean value of the stiffness. The first is slightly too stiff, the latter is too soft and therefore resulting on the safe side. With a total of twenty four beam elements, the results are reproduced adequately. However, the obtained solution deviates from the reference. The reason for that is the fact, that for second order effects, Rubin has taken an unfavourable constant normal force of $-520.3\ kN$ for the whole column. If that effect is accounted for, the results obtained with twenty four elements are:

Table 4: Results with Constant Normal Force

Number of Elements		ϕ_E [$mrad$]	w_E [mm]	M_A [$MN\,m$]
-	Ref. [1]	67.70	423.50	−1.100
24	ASE	67.657	423.27	−1.0995
	STAR2	67.918	424.73	−1.1002

In the case where the example is calculated with DYNA, where a constant normal force of $-520.3kN$ is considered as a primary load case, leading to linearised second order theory and therefore satisfying Rubin's assumption, the results converge to the reference solution. The results, calculated with DYNA and twenty four elements, are presented in Table 5. Furthermore, different single loadings are examined and the results are given in Table 6.

Table 5: Results with DYNA

Number of Elements		ϕ_E [$mrad$]	w_E [mm]	u_E [mm]	M_A [$MN\,m$]	N_A [MN]	N_E [MN]
-	Ref. [1]	67.70	423.50	−1.9050	−1.10	−0.5203	−0.50
24	DYNA	67.6569	423.2766	−1.9045	−1.0994	−0.5203	−0.50

Table 6: Results with DYNA for Combination of Constant Normal Force and Single Loadings

Load Case	ϕ_E [$mrad$]	w_E [mm]	M_A [$MN\,m$]
H^*, H	23.6779	148.3151	−0.3972
q	22.5896	163.3797	−0.6130
ΔT	14.8218	77.1333	−0.0401
ψ^0	2.6481	15.9532	−0.0395
w^0	3.9194	18.4953	−0.0096
$\sum$	67.6569	423.2766	−1.0994

5 Literature

[1] H. Rubin. “Ein einheitliches, geschlossenes Konzept zur Berechnung von Stäben mit stetig verändlichem Querschnitt nach Theorie I. und II. Ordnung”. In: *Bauingenieur 66* (1991), pp. 465–477.