Benchmark Example No. 3

Beam Stresses and Deflections

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VERiFiCATiON
BE3 Beam Stresses and Deflections

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Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT
Procedure(s):	
Topic(s):	
Module(s):	ASE
Input file(s):	rect_beam.dat, I_beam.dat

1 Problem Description

A rectangular beam is supported as shown in Fig. 1 and loaded on the overhangs by a uniformly distributed load q. Determine the maximum bending stress σ in the middle portion of the beam and the deflection δ at the middle of the beam.
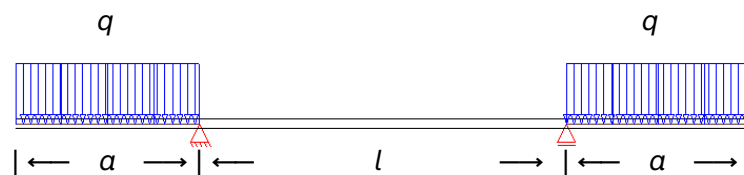

Figure 1: Beam structure

2 Reference Solution

The magnitude of the stresses at a cross-section is defined by the magnitude of the shearing force and bending moment at that cross-section. Under pure bending, the maximum tensile and compressive stresses occur in the outermost fibers. For any cross-section, which has its centroid at the middle of the depth h, and for a linear elastic material behaviour, the maximum stresses occur for $z = \pm h/2$ [1]:

$$\sigma_{max} = \frac{Mh}{2I} \qquad \text{and} \qquad \sigma_{min} = -\frac{Mh}{2I}, \tag{1}$$

in which I, is the moment of inertia of the cross-section with respect to the neutral axis and M the bending moment. For a beam overhanging equally at both supports with a uniformly distributed load applied at the overhangs (Fig. 1), assuming Bernoulli beam theory, the deflection at the middle of the beam is:

$$\delta = \frac{qa^2l^2}{16EI} = \frac{Ml^2}{8EI}, \tag{2}$$

where q is the value of the uniformly distributed load, a the length of the overhangs, l the length of the middle span and M the bending moment at the middle of the beam.

3 Model and Results

The model is analysed for two different cross-sections, a rectangular and a general I-beam cross-section. The properties are defined in Table 1. The results are presented in Table 2. As to be expected, the analysis yields the same results for the maximum bending stress and deflection at the middle of the

beam for the two models. Figure 2 shows the distribution of the stresses along the cross-sections for the two analysed examples. Figure 3 shows the deformed structure with the nodal displacements.

Table 1: Model Properties

Material Properties	Geometric Properties Rectangular	Geometric Properties I-beam	Loading
$E = 30000\,MPa$	$l = 200\,mm$	$l = 200\,mm$	$q = 10\,kN/m$
	$a = 100\,mm$	$b = 16\,mm$	
	$h = 30\,mm$	$t_{web} = 2.174\,mm$	
	$b = 7\,mm$	$t_{flange} = 2\,mm$	
	$I_y = 1.575\,cm^4$	$I_y = 1.575\,cm^4$	

Table 2: Results

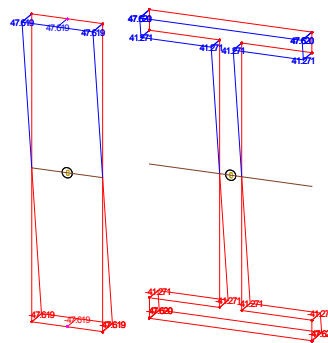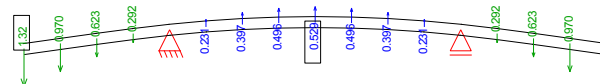|	Rectangular	I-beam	Ref.
σ_{max} [MPa]	47.619	47.620	47.619
δ [mm]	0.529	0.529	0.529

Figure 2: Distribution of stresses

Figure 3: Deformed Structure

4   Conclusion

This example adresses the computation of beam stresses and deflections. It has been shown that the behaviour of the beam is captured with an excellent accuracy.

5 Literature

[1] S. Timoshenko. *Strength of Materials, Part I, Elementary Theory and Problems*. 2nd. D. Van Nostrand Co., Inc., 1940.