Benchmark Example No. 28

Cylindrical Hole in an Infinite Elastic Medium

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VERiFiCATiON
BE28 Cylindrical Hole in an Infinite Elastic Medium

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Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview	
Element Type(s):	C2D
Analysis Type(s):	STAT
Procedure(s):	
Topic(s):	SOIL
Module(s):	TALPA
Input file(s):	hole_elastic.dat

1 Problem Description

This problem consists of a cylindrical hole in an infinite elastic medium subjected to a constant in-situ state, as shown in Fig. 1. The material is assumed to be isotropic and elastic. The stresses and the displacements are verified.

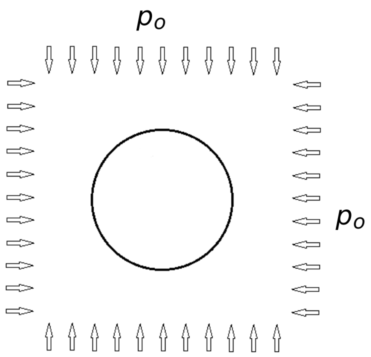

Figure 1: Problem Description

2 Reference Solution

The problem of calculating the displacements and stresses outside a circular hole in an infinite elastic medium, with a uniform stress state far from the hole, was first solved by the German engineer Kirsch in 1898. It is a rather important topic due to the fact that most of the holes drilled through rock are of circular section.
The classical Kirsch solution can be used to find the radial and tangential displacement fields and stress distributions, for a cylindrical hole in an infinite isotropic elastic medium under plane strain conditions. The stresses σ_r and σ_θ for a point at polar coordinates (r, θ) outside the cylindrical opening of radius α are given by [1]:

$$\sigma_r = \frac{p_1 + p_2}{2}\left(1 - \frac{\alpha^2}{r^2}\right) + \frac{p_1 - p_2}{2}\left[1 - \frac{4\alpha^2}{r^2} + \frac{3\alpha^4}{r^4}\right] cos\, 2\theta \tag{1}$$

$$\sigma_\theta = \frac{p_1 + p_2}{2}\left(1 + \frac{\alpha^2}{r^2}\right) - \frac{p_1 - p_2}{2}\left(1 + \frac{3\alpha^4}{r^4}\right) cos\, 2\theta \tag{2}$$

The radial outward displacement u_r, assuming conditions of plane strain, is given by:

$$u_r = \frac{p_1 + p_2}{4G}\frac{\alpha^2}{r} + \frac{p_1 - p_2}{4G}\frac{\alpha^2}{r}\left[4(1-\nu) - \frac{\alpha^2}{r^2}\right] cos\, 2\theta \tag{3}$$

where G is the shear modulus, ν the Poisson ratio and p_1, p_2, θ, r are defined in Fig. 2

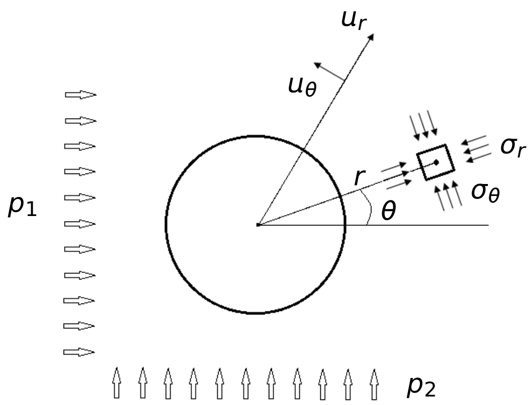

Figure 2: Cyllindrical Hole in an Infinite Elastic Medium

3 Model and Results

The properties of the model are defined in Table 1. The radius of the hole is $1\ m$ and is assumed to be small compared to the length of the cylinder, therefore $2D$ plane strain conditions are in effect. A fixed external boundary is located $29.7\ m$ from the hole center. The model is presented in Fig. 3. The stresses and displacements are calculated and verified with respect to the formulas provided in Section 2.

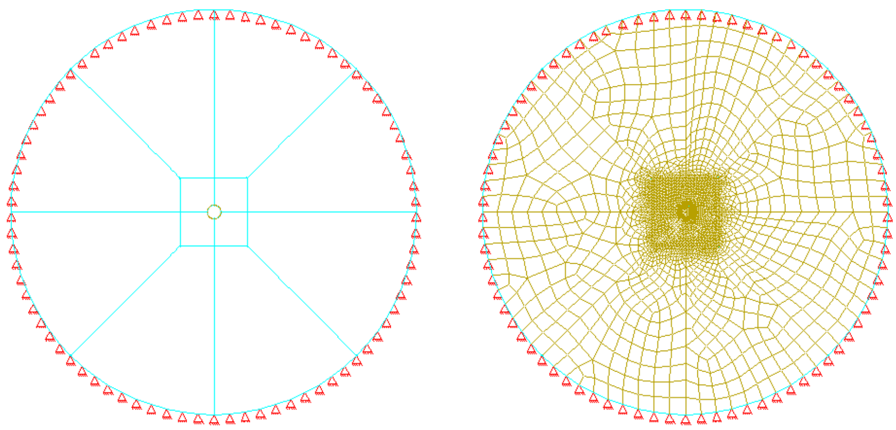

Figure 3: Finite Element Model

Table 1: Model Properties

Material Properties	Geometric Properties	Pressure Properties
$E = 6777.9\ MPa$	$\alpha = 1\ m$	$P_o = 30\ MPa$

Table 1: (continued)

Material Properties	Geometric Properties	Pressure Properties
$\nu = 0.21$	$r_{boundary} = 29.7\ m$	

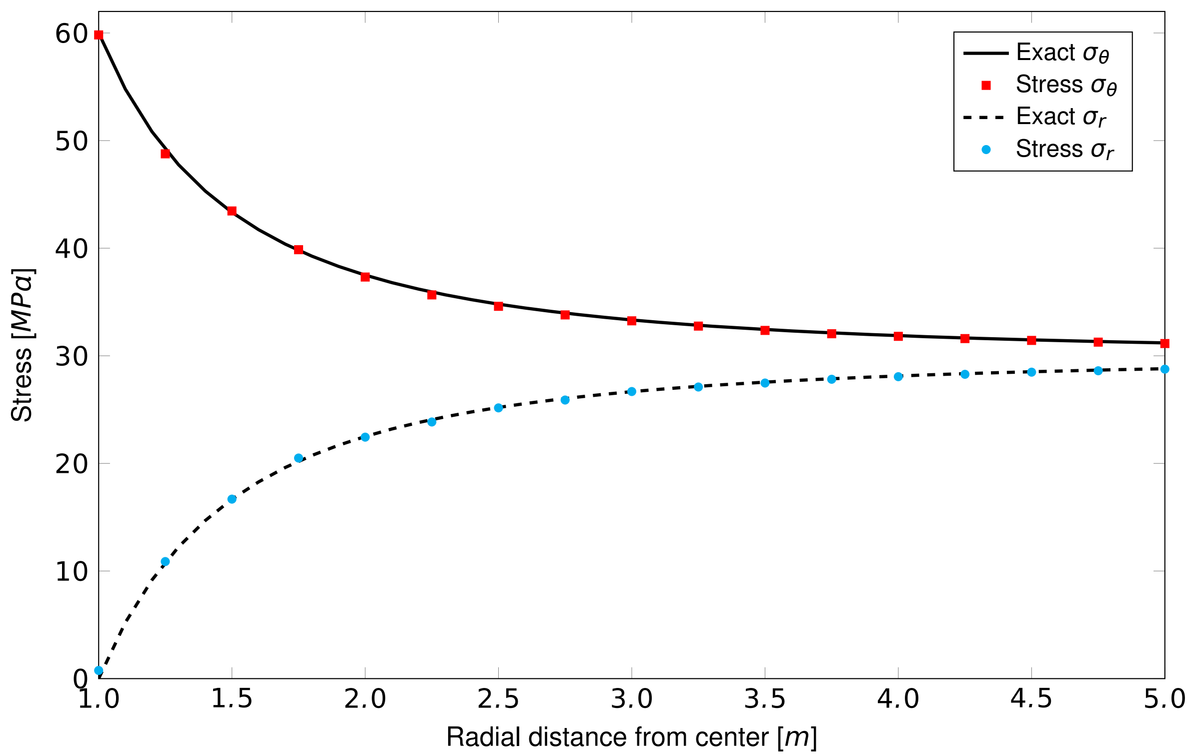

Figure 4: Radial and Tangential Stresses for Cylindrical Hole in Infinite Elastic Medium

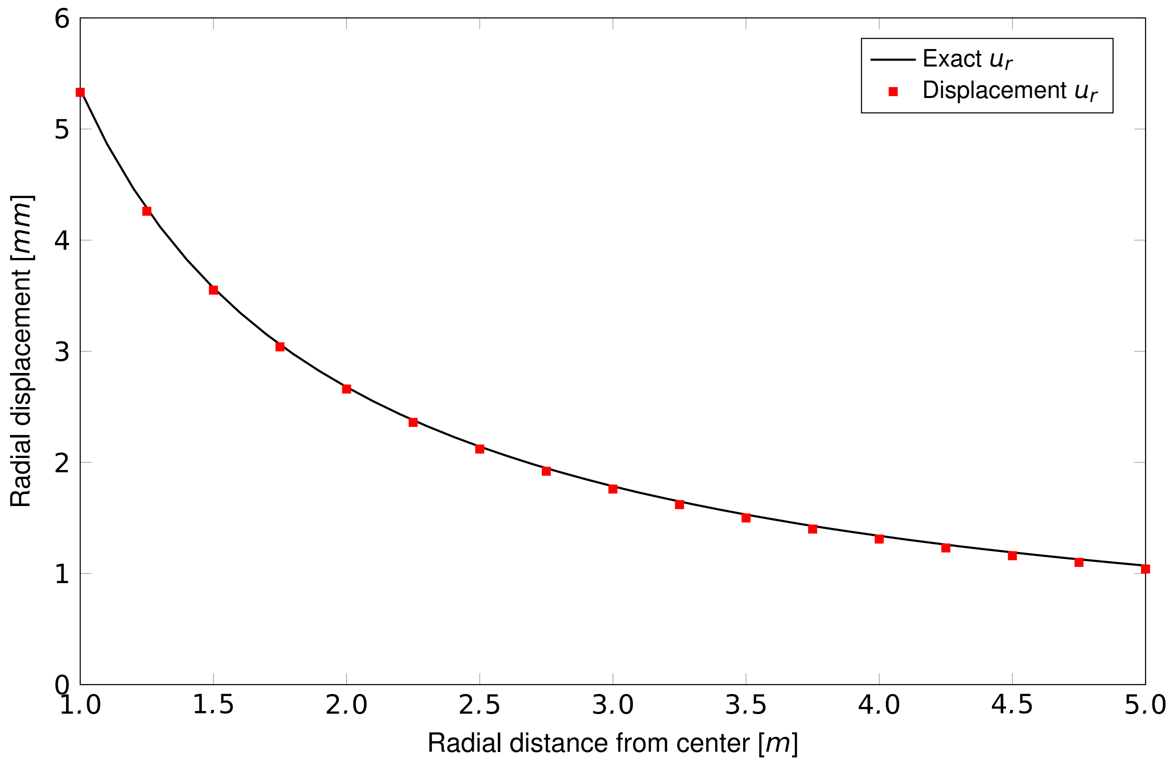

Figure 5: Radial Displacement for Cylindrical Hole in Infinite Elastic Medium

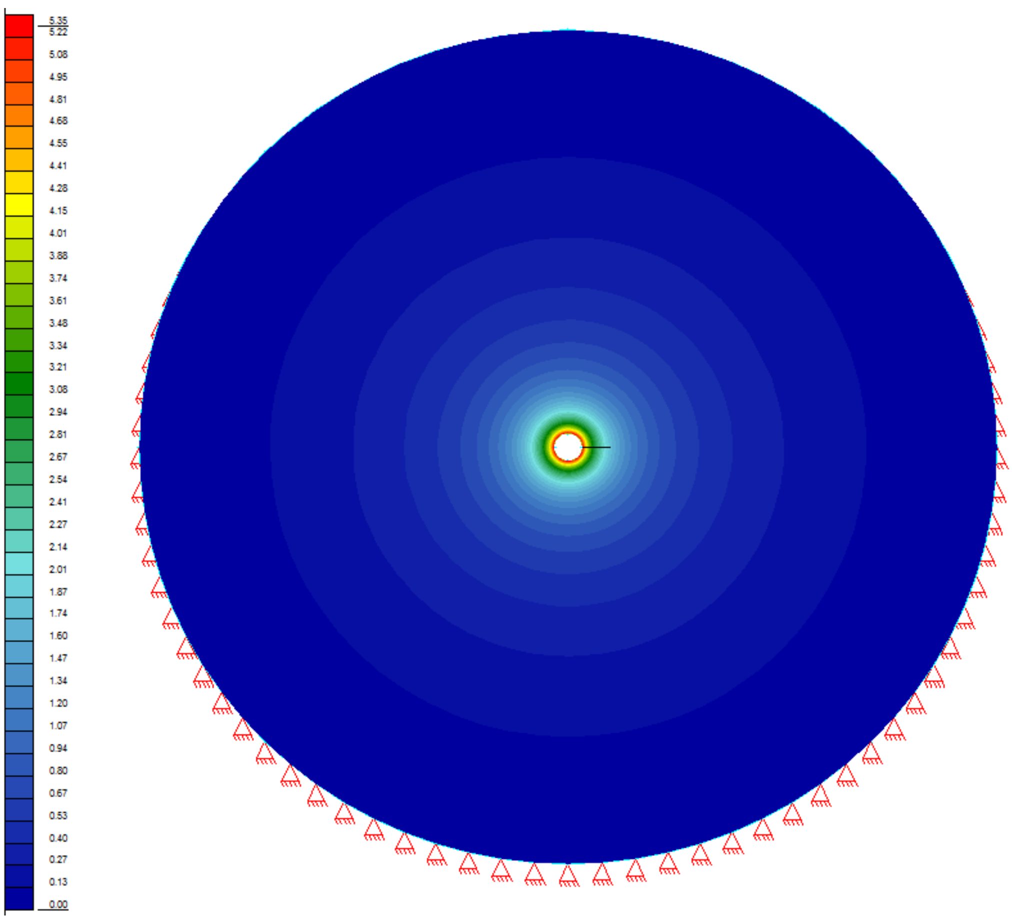

Figure 6: Total Displacement Distribution

Figures 4 and 5 show the radial and tangential stress and the radial displacement respectively, along a line, lying on the X-axis. This line (cut) can be visualised in Fig. 6, where the radial displacement distribution is illustrated. The results are in very good agreement with the reference solution.

4 Conclusion

This example verifies the deformation and stresses behaviour of a cylindrical hole in an infinite elastic medium. It has been shown that the behaviour of the model is captured accurately.

5 Literature

[1] J. C. Jaeger, N. G. W. Cook, and R. W. Zimmerman. *Fundamentals of Rock Mechanics*. 4th. Blackwell Publishing, 2007.