



Benchmark Example No. 25

## Eigenvalue Analysis of a Beam Under Various End Constraints

**VERiFiCATION**  
**BE25 Eigenvalue Analysis of a Beam Under Various End Constraints**

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The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

**Front Cover**

Arnulfsteg, Munich Photo: Hans Gössing

### Overview

<b>Element Type(s):</b>	B3D
<b>Analysis Type(s):</b>	DYN
<b>Procedure(s):</b>	EIGE
<b>Topic(s):</b>	
<b>Module(s):</b>	DYNA
<b>Input file(s):</b>	<a href="#">eigenvalue_analysis.dat</a>

## 1 Problem Description

This problem consists of a beam with various end constraints, as shown in Fig. 1. The eigenfrequencies of the the system are determined and compared to the exact reference solution for each case.

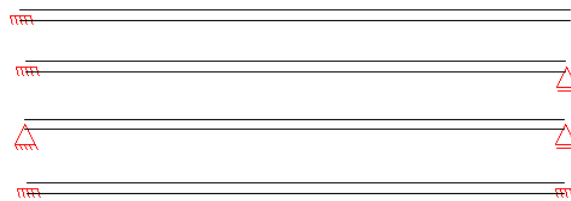


Figure 1: Problem Description

## 2 Reference Solution

The general formula to determine the eigenfrequency of a standard Bernoulli beam for a linear elastic material is given by [1] [2]

$$f = \frac{\lambda^2}{2\pi} \sqrt{\frac{EI}{\mu l^4}} \quad (1)$$

where  $EI$  the flexural rigidity of the beam,  $l$  the length,  $\mu = \gamma * A/g$  the mass allocation and  $\lambda$  a factor depending on the end constraints. The values of  $\lambda$  for various cases are given in Table 1. In this example, we analyse four different cases of a beam structure:

1. simple cantilever
2. cantilever with simply supported end
3. simply supported
4. both ends fixed

Table 1: Constraints Factor

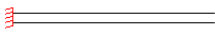
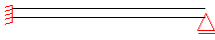

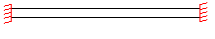
End Constraints	$\lambda$
	$\lambda = 1.875$

Table 1: (continued)

End Constraints	$\lambda$
	$\lambda = 3.926$
	$\lambda = \pi$
	$\lambda = 4.73$

### 3 Model and Results

The properties of the model are defined in Table 2 and the resulted eigenfrequencies are given in Table 3. For the eigenvalue analysis a consistent mass matrix formulation is used as well as a Bernoulli beam. The finite element model for all examined cases consists of ten beam elements.

Table 2: Model Properties

Material Properties	Geometric Properties
$E = 200 \text{ MPa}$	$h = 1 \text{ cm}, b = 1 \text{ cm}, l = 1 \text{ m}$
$\gamma = 25 \text{ kN/m}^3$	$A = 1 \text{ cm}^2, I = 0.1 \text{ cm}^4, \mu = 0.025 \text{ t/m}$

Table 3: Results

	Eigenfrequency	SOF. [Hz]	Ref. [Hz]
simple cantilever		0.457	0.457
cantilever with simply supported end		2.004	2.003
simply supported		1.283	1.283
both ends fixed		2.907	2.907

### 4 Conclusion

The purpose of this example is to test the eigenvalue capability of the program w.r.t. different options. It has been shown that the eigenfrequencies for all beam systems are calculated accurately.

### 5 Literature

- [1] K. Holschemacher. *Entwurfs- und Berechnungstabellen für Bauingenieure*. 3rd. Bauwerk, 2007.
- [2] S. Timoshenko. *Vibration Problems in Engineering*. 2nd. D. Van Nostrand Co., Inc., 1937.