SOFiSTiK

Benchmark Example No. 23

Undamped Free Vibration of a SDOF System

SOFiSTiK | 2022

VERiFiCATiON
BE23 Undamped Free Vibration of a SDOF System

VERiFiCATiON Manual, Service Pack 2022-12 Build 74

SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview

Element Type(s):	SPRI
Analysis Type(s):	DYN
Procedure(s):	TSTP
Topic(s):	
Module(s):	DYNA
Input file(s):	undamped_sdof.dat

1 Problem Description

This problem consists of an undamped linearly elastic SDOF system undergoing free vibrations, as shown in Fig. 1. The response of the system is determined and compared to the exact reference solution.

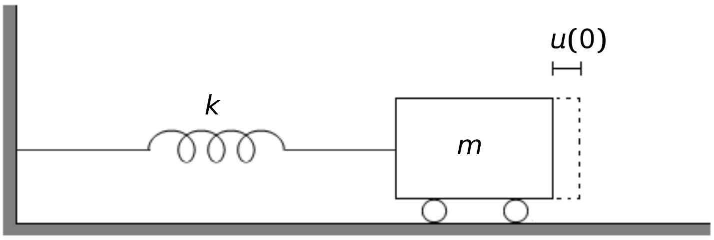

Figure 1: Problem Description

2 Reference Solution

The essential physical properties of a linearly elastic structural system subjected to an external excitation or dynamic loading are its mass, stiffness and damping. In the simplest model of a SDOF system, as shown in Fig. 2 in its idealized form, these properties are concentrated in a single physical element. For this system the elastic resistance to displacement is provided by the spring of stiffness k, while the energy-loss mechanism by the damper c. The mass m is included in the rigid body, which is is able to move only in simple translation, and thus the single displacement coordinate $u(t)$ completely describes its position [1].

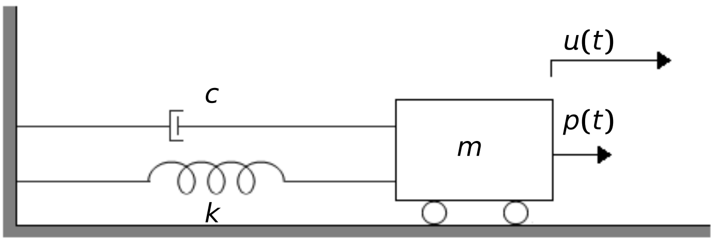

Figure 2: Problem Description

The motion of a linear SDOF system, subjected to an external force $p(t)$ is governed by [1] [2]:

$$m\ddot{u} + c\dot{u} + ku = p(t) \tag{1}$$

Setting $p(t) = 0$, gives the differential equation governing the free vibration of the system

$$m\ddot{u} + c\dot{u} + ku = 0 \tag{2}$$

For a system without damping (c =0), Eq. 2 specialises to

$$m\ddot{u} + ku = 0 \tag{3}$$

Free vibration is initiated by disturbing the system from its static equilibrium position by imparting the mass some displacement $u(0)$ and/or velocity $u(\dot{0})$ at time zero. Subjected to these initial conditions, the solution to the homogeneous differential equation of motion is:

$$u(t) = u(0)\,cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n} sin(\omega_n t) \tag{4}$$

where

$$\omega_n = \sqrt{\frac{k}{m}} \tag{5}$$

represents the natural circular frequency of vibration and f the natural cyclic frequency of vibration

$$f_n = \frac{\omega_n}{2\pi} \tag{6}$$

The period T represents the time required for the undamped system to complete one cycle of free vibration and is given by

$$T_n = \frac{2\pi}{\omega_n} = \frac{1}{f_n} \tag{7}$$

3 Model and Results

The properties of the model are defined in Table 1. The system is initially disturbed from its static equilibrium position by a displacement of $20\ mm$ and is then let to vibrate freely. Eq. 4 is plotted in Fig.

4, presenting that the system undergoes vibration motion about its undeformed ($u = 0$) position, and that this motion repeats itself every $2\pi/\omega_n$ seconds. The exact solution is compared to the calculated time history of the displacement of the SDOF system for different time integration methods. The time step is taken equal to $0.02\ sec$ corresponding to a dt/T ratio of $1/50$.

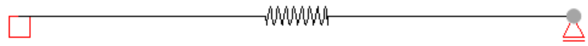

Figure 3: Finite Element Model

Table 1: Model Properties

Model Properties	Excitation Properties
$m = 1\,t$	$u(0) = 20\,mm$
$k = 4\pi^2\,kN/m$	$\dot{u}(0) = 0$
$T = 1\,sec$	

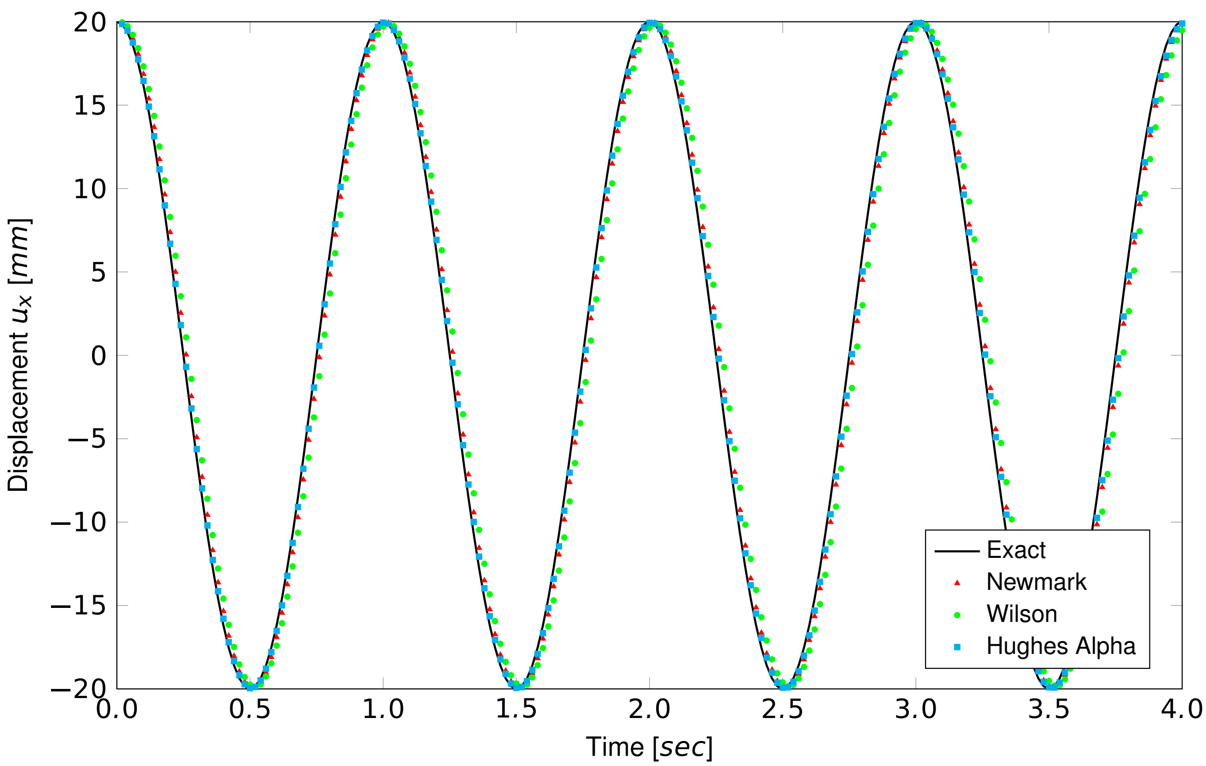

Figure 4: Undamped Free Vibration Response

From the results presented in Table 2, we observe that the response computed by the examined integration schemes is in a good agreement with the exact solution.

Table 2: Results

	Integration method	Newmark	Wilson	Hughes Alpha	Ref.
u_{max} $[mm]$		19.949	19.963	19.956	20.000

4 Conclusion

This example examines the response of a linear elastic undamped SDOF system undergoing free vibration. It has been shown that the behaviour of the system is captured adequately.

5 Literature

[1] R. W. Clough and J. Penzien. *Dynamics of Structures*. 3rd. Computers & Structures, Inc., 2003.
[2] A. K. Chopra. *Dynamics of Structures: Theory and Applications to Earthquake Engineering*. Prentice Hall, 1995.