Benchmark Example No. 2

Kinematic Coupling Conditions

SOFiSTiK | 2022

VERiFiCATiON
BE2 Kinematic Coupling Conditions

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Front Cover

Arnulfsteg, Munich Photo: Hans Gössing

Overview	
Element Type(s):	COUP
Analysis Type(s):	STAT
Procedure(s):	
Topic(s):	
Module(s):	SOFiMSHC, ASE
Input file(s):	coupling.dat

1 Problem Description

This problem verifies the kinematic coupling conditions for a structural point. Each coupling condition is tested on a pair of beams coupled with each other through structural points, as shown in Fig. 1. Four different cases are considered and the deflections of the beams are determined and compared to the analytical solution.

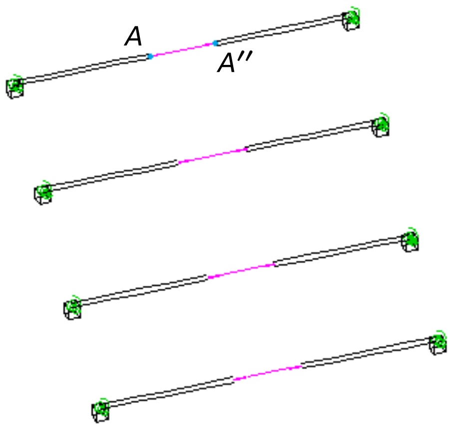

Figure 1: Problem Description

2 Reference Solution

In this example the problem of coupling structural points is treated. Through the definition of kinematic coupling conditions between structural points the constraint of one or multiple degrees of freedom is allowed. The displacement values of the given structural point A'' are defined according to the respective displacement values of the referenced (or master-) node A. Various cases are possible in SOFiSTiK for the coupling conditions. With the exception of the three conditions KPX, KPY and KPZ, which only couple the corresponding displacement. e.g. $u_x = u_{xo}$, all other coupling conditions satisfy the mechanical equilibrium conditions by taking the real distances between the two connected points into account, e.g. the conditions $KPPX$, $KPPY$, $KPPZ$ correspond to the following expressions respectively [1] [2]:

$$u_x = u_{xo} + \phi_{yo}(z - z_o) - \phi_{zo}(y - y_o) \tag{1}$$

$$u_y = u_{yo} + \phi_{zo}(x - x_o) - \phi_{xo}(z - z_o) \tag{2}$$

$$u_z = u_{zo} + \phi_{xo}(y - y_o) - \phi_{yo}(x - x_o) \quad (3)$$

Mechanically they act like infinitely stiff structural members. A number of additional literals are provided in SOFiSTiK which allow to define a combination of coupling relations. For example, a rigid connection with hinged conditions at the reference node is described by

$$KP = KPPX + KPPY + KPPZ \quad (4)$$

whereas

$$KF = KP + KMX + KMY + KMZ = KPPX + KPPY + KPPZ + KMX + KMY + KMZ \quad (5)$$

describes mechanically a rigid connection with clamped support at the reference node. Further information on the topic are provided in SOFiSTiK manual of module SOFiMSHC [1].

3 Model and Results

The general properties of the model are defined in Table 1. All beams are of 4 m length and consist of a standard rectangular cross-section and a standard concrete material. The structural points A and A'' have a distance of 2 m in the axial direction. Four coupling conditions are considered :

- $KPPX$, where only the displacement in the global x direction is connected
- LPX, where only the displacement in the structural point's local x direction is connected
- KP, where the displacements in x, y and z direction are connected
- KF, where the displacements and the rotations in x, y and z direction are connected

All cases are tested for four loadcases, i.e. a horizontal P_y, a longitudinal P_x, a vertical P_z and a rotational M_x.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\,30/45$	$l_{beam} = 4\,m$	$P_y = 50.0\,kN$
	$h_A = 0.4\,m$, $b_A = 0.2\,m$	$P_x = -50.0\,kN$
	$h_{A''} = 0.3\,m$, $b_{A''} = 0.15\,m$	$P_z = 50.0\,kN$
	$(x_{A''} - x_A) = 2\,m$	$M_x = 10.0\,kN$
	$(y_{A''} - y_A) = 0\,m$	
	$(z_{A''} - z_A) = 0\,m$	

In the cases, where only a displacement is transferred in the vertical u_z or horizontal direction u_y , a rotation results in the other direction. If for example, we consider a coupling of only the displacement in the y direction, then a rotation of $\phi_z = 3u_y/(2l_{beam})$ will also result as the effect of a prescribed displacement of value u_y at the beam tip A''.

Table 2: Results for KPPX Coupling Condition

Load	u_x [mm]	
Case	SOF.	Ref.
P_x	−0.107	−0.107

Table 3: Results for LPX Coupling Condition

Load	u_y [mm]		ϕ_z [mrad]	
Case	SOF.	Ref.	SOF.	Ref.
P_y	165.284	165.284	−61.919	−61.982

Table 4: Results for KP and KF Coupling Condition

Coupling		KP			KF			
DOF / LC		P_y	P_x	P_z	P_y	P_x	P_z	M_x
u_x [mm]	SOF.	0.0	−0.107	0.0	0.0	−0.107	0.0	0.0
	Ref.	0.0	−0.107	0.0	0.0	−0.107	0.0	0.0
u_y [mm]	SOF.	165.284	0.0	0.0	49.325	0.0	0.0	0.0
	Ref.	165.284	0.0	0.0	49.325	0.0	0.0	0.0
u_z [mm]	SOF.	0.0	0.0	41.528	0.0	0.0	12.531	0.0
	Ref.	0.0	0.0	41.528	0.0	0.0	12.531	0.0
ϕ_x [mrad]	SOF.	0.0	0.0	0.0	0.0	0.0	0.0	6.671
	Ref.	0.0	0.0	0.0	0.0	0.0	0.0	6.671
ϕ_y [mrad]	SOF.	0.0	0.0	15.510	0.0	0.0	0.200	0.0
	Ref.	0.0	0.0	15.573	0.0	0.0	0.200	0.0
ϕ_z [mrad]	SOF.	−61.919	0.0	0.0	−0.725	0.0	0.0	0.0
	Ref.	−61.982	0.0	0.0	−0.725	0.0	0.0	0.0

The results are presented in Tables 2 - 4, where they are compared to the reference results calculated with the formulas provided in Section 2. Due to the extent of the results only non zero values will be presented in the result tables. Figures 2, 3 present the results for the *KF* coupling condition for the load cases 1 to 4 for both displacements and rotations, respectively.

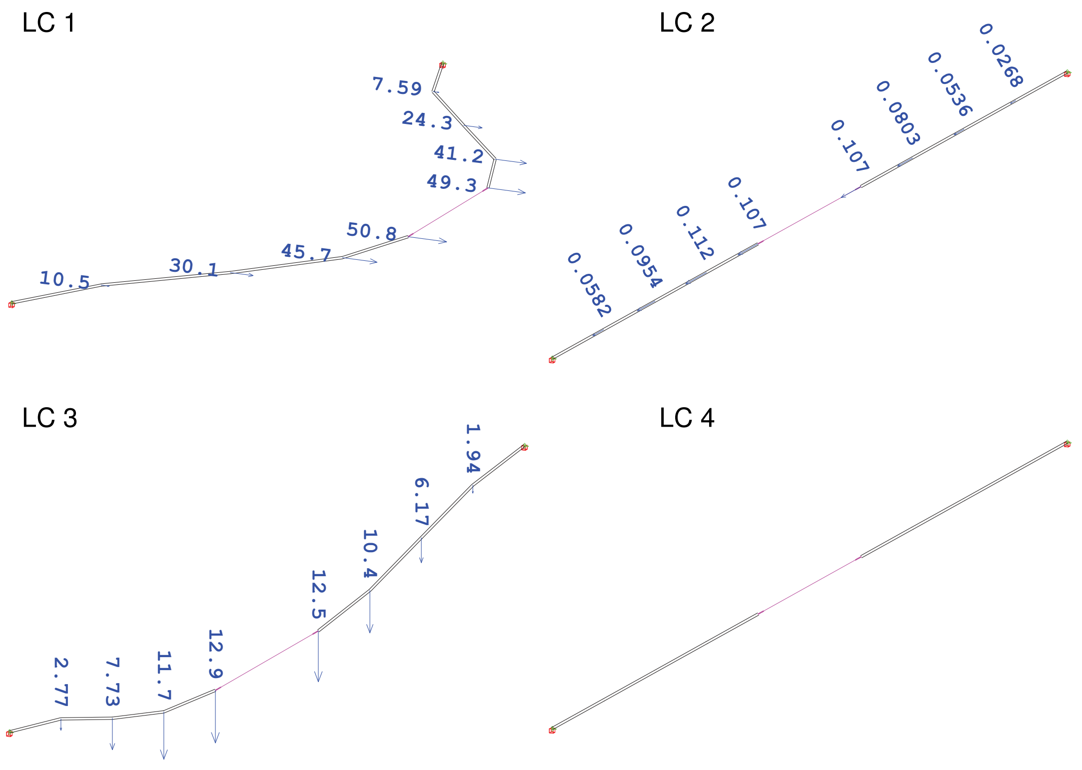

Figure 2: Displacement Results for KF coupling for LC 1-4

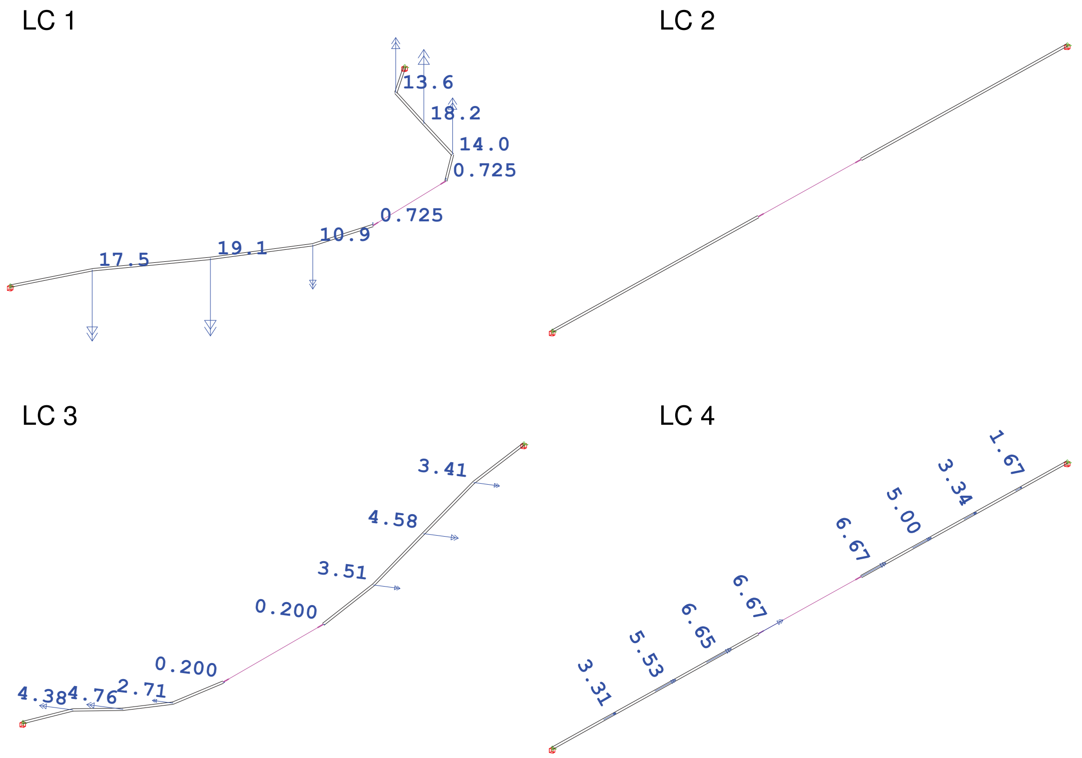

Figure 3: Rotation Results for KF coupling for LC 1-4

4 Conclusion

This example verifies the coupling of structural points. It has been shown that the behaviour is accurately captured.

5 Literature

[1] *SOFiMSHC Manual: Geometric Modelling*. Version 18-0. SOFiSTiK AG. Oberschleißheim, Germany, 2017.

[2] *SOFiMSHA Manual: Import and Export of Finite Elements and Beam Structures*. Version 18-0. SOFiSTiK AG. Oberschleißheim, Germany, 2017.