Benchmark Example No. 12

Cantilever in Torsion

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VERiFiCATiON
BE12 Cantilever in Torsion

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Front Cover

Arnulfsteg, Munich Photo: Hans Gössing

Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT, GNL
Procedure(s):	
Topic(s):	
Module(s):	ASE
Input file(s):	torsion.dat

1 Problem Description

The problem consists of a cantilever beam as shown in Fig. 1. The tip of the cantilever is offsetted in y-direction by $\Delta_y = l/200 = 2.5\ cm$, creating a geometrical imperfection. The beam is loaded with a transverse force P_z and an axial force P_x. The imperfection acts as a lever arm for the loading, causing a torsional moment. The torsional moment at the support with respect to the local and global coordinate system is determined.

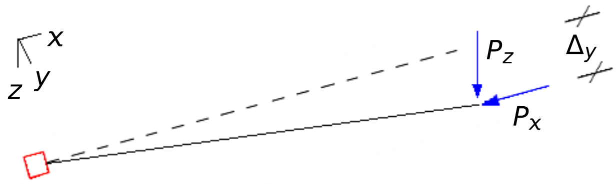

Figure 1: Problem Description

2 Reference Solution

In order to account for the effect of the geometrical imperfection on the structure, second-order theory should be used, where the equilibrium is established at the deformed system. According to the equilibrium of moments at the deformed system, with respect to the global x-axis, the torsional moment at the support $M_{x_{global}}$ is:

$$M_{x_{global}} = P_z\ (u_y + \Delta_y) - P_y\ u_z\ , \tag{1}$$

whereas by the local x-axis the torsional moment $M_{x_{local}}$ is:

$$M_{x_{local}} = P_z\ u_y + P_x\ (\frac{\Delta_y}{l})\ u_z\ , \tag{2}$$

where l is the length of the beam, Δ_y the initial geometrical imperfection and P_x is negative for compression.

3 Model and Results

The properties of the model [1] [2] are defined in Table 1. A standard steel material is used as well as a standard hot formed hollow section with properties according to DIN 59410, DIN EN 10210-2. A safety factor $\gamma_M = 1.1$ is used, which according to DIN 18800-2 it is applied both to the yield strength and the stiffness. Furthermore, the self weight, the shear deformations and the warping modulus C_M are neglected. At the support the warping is not constrained.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$S\,355$	$l = 5\,m$	$P_z = 10\,kN$
$\gamma_M = 1.1$	$RRo/SH\ 200 \times 100 \times 10$ [3]	$P_x = 100\,kN$
$C_M = 0$	$\Delta_y = 2.5\,cm$	

Table 2: Results

	u_y $[cm]$	u_z $[cm]$	$M_{x_{global}}$ $[kNcm]$	$M_{x_{local}}$ $[kNcm]$	P_{buck} $[kN]$
SOF.	3.209	10.204	57.08	26.98	163.7
Ref.[4]	3.20	10.2	57.0	26.9	164

The corresponding results are presented in Table 2. Figure 2 shows the deformed shape of the structure and the nodal displacements for the z and y direction. From the presented results, we can observe that the values of the moments are correctly computed. Here has to be noted that the reference results are according to [1], where they are computed with another finite element software, and not with respect to an analytical solution.

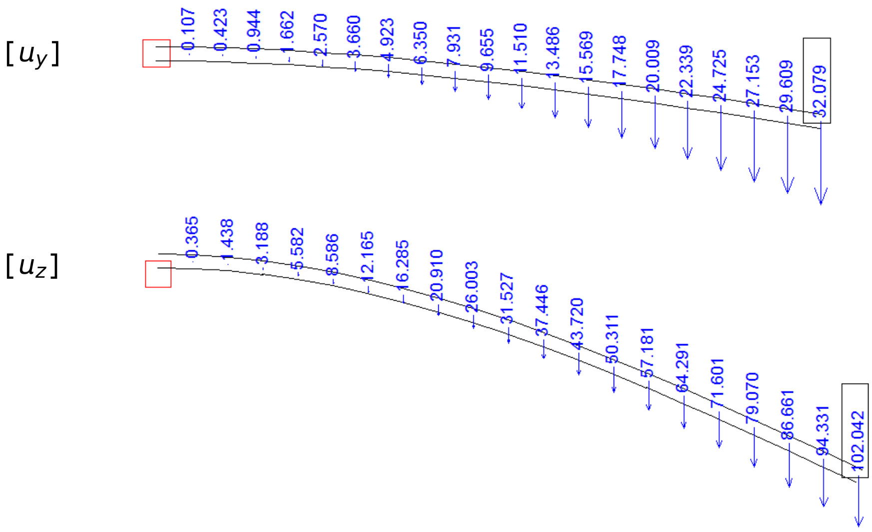

Figure 2: Deformations $[mm]$

4 Conclusion

This example presents a case where torsion is induced to the system because of an initial geometrical imperfection. It has been shown that the behaviour of the beam is captured accurately.

5 Literature

[1] V. Gensichen and G. Lumpe. *Zur Leistungsfähigkeit, korrekten Anwendung und Kontrolle räumlicher Stabwerksprogramme*. Stahlbau Seminar 07.

[2] V. Gensichen. *Zur Leistungsfähigkeit räumlicher Stabwerksprogramme, Feldstudie in Zusammenarbeit mit maßgebenden Programmherstellern*. Stahlbau Seminar 07/08.

[3] K. Holschemacher. *Entwurfs- und Berechnungstafeln für Bauingenieure*. 3rd. Bauwerk, 2007.

[4] V. Gensichen and G. Lumpe. “Zur Leistungsfähigkeit, korrekten Anwendung und Kontrolle von EDV-Programmen für die Berechnung räumlicher Stabwerke im Stahlbau”. In: *Stahlbau 77 (Teil 2)* (2008).