Benchmark Example No. 11

Plastification of a Rectangular Beam

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VERiFiCATiON
BE11 Plastification of a Rectangular Beam

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SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Arnulfsteg, Munich   Photo: Hans Gössing

Overview

Element Type(s): B3D, BF2D, SH3D
Analysis Type(s): STAT, MNL
Procedure(s): LSTP
Topic(s):
Module(s): ASE, STAR2, TALPA
Input file(s): beam_star2.dat, fiber_beam.dat, quad.dat

1 Problem Description

The problem consists of a rectangular cantilever beam, loaded in pure bending as shown in Fig. 1. The model [1] is analysed for different load levels, including the capacity limit load, where the cross-section fully plastifies. The beam is modelled and analysed with different elements and modules.

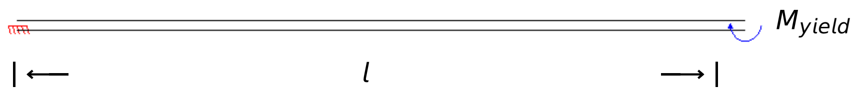

Figure 1: Problem Description

2 Reference Solution

The model follows an elastic-perfectly-plastic stress-strain behaviour as shown in Fig. 2. Under this assumption, the beam remains elastic until the outermost fibers reach the yield stress. The corresponding limit load can be calculated as:

$$M_{yield} = \frac{\sigma_{yield}\, bh^2}{6}, \quad (1)$$

where σ_{yield} is the yield stress, b and h the dimensions of the beam. The cross-section fully plastifies when the load reaches $M = M_{ult} = 1.5 \times M_{yield}$, where all fibers of the beam are in condition of yielding [2].

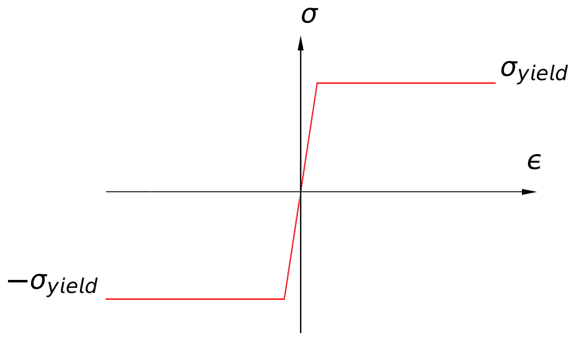

Figure 2: Stress-Strain Curve

3 Model and Results

The properties of the model are defined in Table 1. A standard steel material is used and modified accordingly to account for the intended elastic-perfectly-plastic material behaviour.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 210000\, MPa$	$L = 1\, m$	$M_{yield} = 280\, Nm$
$\nu = 0.3$	$h = 20\, mm$	
$\sigma_{yield} = 420\, MPa$	$b = 10\, mm$	

The structure is modelled and analysed in various ways. For the first case the fiber beam is used (TALPA), where the cross-section is discretised into single fibers and directly integrates the continuum mechanical material reaction into beam theory, and physically nonlinear analysis is performed. For the second case the standard beam elements are used and the model is analysed with STAR2 where a nonlinear stress and strain evaluation determination is performed. For the third case, the quad elements are used and a nonlinear analysis is done with ASE. The results are presented in Table 2 for the three cases.

Table 2: Results

M/M_{yield}	Fiber Beam	Standard Beam	Quad		Ref.
	σ [MPa]	σ [MPa]	σ	σ_{eff}	
0.99	415.80	415.80	415.80	415.80	$\sigma < 420.00$ Fully Elastic
1.00	≤ 420	≤ 420	≤ 420	≤ 420	$\sigma \leq 420.00$ First Yield
1.48	≤ 420	≤ 420	≤ 431.0	≤ 420	$\sigma \leq 420.00$ Elastic-Plastic
1.50	Fully-Plastic	Fully-Plastic No Convergence	Fully-Plastic No Convergence		$\sigma = 420.00$ Fully-Plastic
1.51	Fully-Plastic No Convergence	Fully-Plastic No Convergence	Fully-Plastic No Convergence		Fully-Plastic No Convergence

This benchmark is designed to test elastic-plastic material behaviour under uniaxial loading conditions. From the above results, it is evident that both beam element formulations adequately reproduce the intended behaviour. Fig. 3 shows the distribution of stresses for the case of the fiber beam with $M/M_{yield} = 0.99$, 1.0 and 1.5. For the quad element, the stress appears to exceed the limit value of $420\, MPa$. This is due to the fact that, as the plasticity involves at the cross-section, plastic strains also appear in the lateral direction. This causes a biaxial stress state, which is not neglected by the quad formulation, as shown in Fig. 4 for $M/M_{yield} = 1.0$ and 1.48. A closer look at the list of results though, reveals that the $\mathtt{effective\ stresses}$ do not exceed the σ_{yield}.

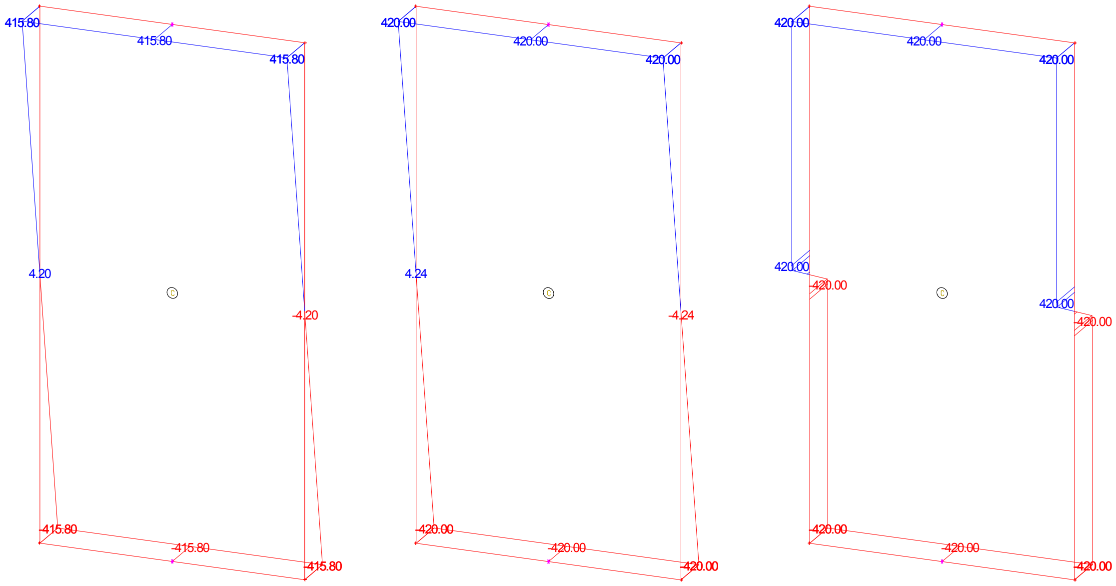

Figure 3: Fiber Beam Stress State

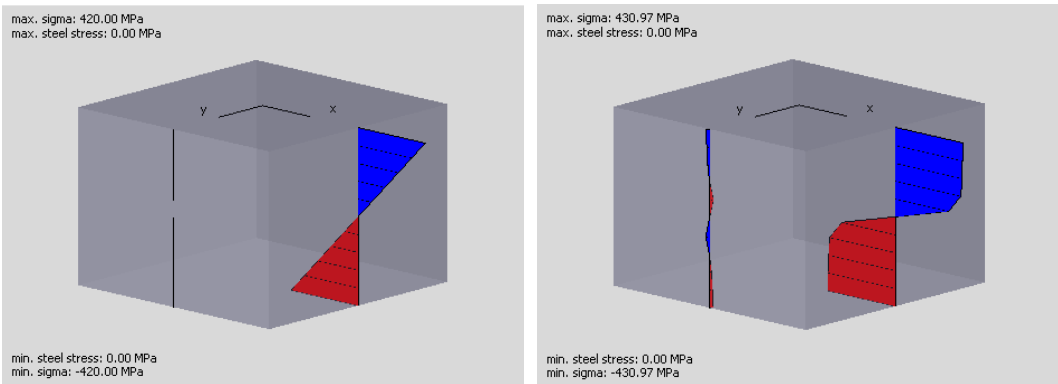

Figure 4: Quad Stress State

4 Conclusion

This example presents the pure bending of beams beyond their elastic limit for a non elastic material. It has been shown that the behaviour of the beam is accurately captured for all three modelling options.

5 Literature

[1] *Verification Manual for the Mechanical APLD Application, Release 12.0*. Ansys, Inc. 2009.
[2] S. Timoshenko. *Strength of Materials, Part II, Advanced Theory and Problems*. 2nd. D. Van Nostrand Co., Inc., 1940.